Exam 3 Solutions

15-122 Principles of Imperative Computation, Summer 1, 2015
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Instructions

- This exam is closed-book with one sheet of notes permitted.
- You have 170 minutes to complete the exam.
- There are 6 problems on 17 pages (including two blank pages at the end).
- Read each problem carefully before attempting to solve it.
- Do not spend too much time on any one problem.
- Consider if you might want to skip a problem on a first pass and return to it later.
- Except for the first question, the entire exam is in C. You can assume the presence of any standard C libraries discussed in class (including `limits.h`, `xalloc.h`, and `contracts.h`) throughout the exam.

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	Max	Score
C0 and C	30	
Requiring Safety	30	
High-Speed Data Structures	50	
Generic Exam Question	40	
Graph Expansion	30	
Minimum Spanning Trees	20	
Total:	200	

1 C0 and C (30 points)

For the C code in this question, you can assume standard implementation-defined behavior.

In every case, assume we are calling `gcc` with the command
"`gcc -Wall -Wextra -Werror -Wshadow -std=c99 -pedantic example.c`"

Assume the following declarations, where "..." refers to an arbitrary unknown value.

In C0

```
int INT_MIN = 0x80000000;

int x = ...;
int y = ...;
int z = ...;

int[] A = alloc_array(int, 5);
int* p = alloc(int);
```

In C

```
#include <limits.h>

unsigned int x = ...;
int y = ...;
int z = ...;

int *A = xcalloc(5, sizeof(int));
int *p = xmalloc(sizeof(int));
```

For the following expressions, indicate all possible outcomes among `true`, `false`, *error* (for program-stopping runtime errors), and *undefined*. If the behavior may be *undefined* then all other outcomes are automatically possible, so you don't need to list them explicitly. Assume that all initializations succeed without error. To get you started we have already filled in first row for you.

Expression	In C0	In C
`x * 2 > x`	`false`, `true`	`false`, `true`
`x * 4 == x << 4`	**true, false**	**true, false**
`x / 8 == x >> 3`	**true, false**	**true**
`y == INT_MIN || y > y - 1`	**true**	**true**
`z == 0 || (y/z)*z + y%z == y`	**true, error**	**undefined**
`z == 0 || (122/z)*z + 122%z == 122`	**true**	**true**
`y + z >= 0 || y + z < 0`	**true**	**undefined**
`y <= 0 || z <= 0 || (y + z)/y <= z + y`	**true, false**	**undefined**
`*p == A[0]`	**true**	**undefined**
`A[0] == A[A[A[0]]]`	**true**	**true**
`A[5] == A[A[A[5]]]`	**error**	**undefined**

2 Requiring Safety (30 points)

For the functions in this question, write (additional) preconditions that are sufficient to ensure that there will be no undefined behavior and no memory leaks. Your preconditions allow the function to run *whenever* undefined behavior and memory leaks would not occur. Make sure it's not possible to cause undefined behavior in the precondition itself!

If it's not possible to ensure that a function is free of undefined behavior and memory leaks, then you can write the precondition `false`, which indicates that the function cannot be run safely. If no preconditions are needed, you can write the precondition `true`.

Do not assume *any* implementation-defined behaviors except that a byte is 8 bits. Your preconditions should make your functions safe for any implementation-defined behaviors.

5 **Task 1**

```
unsigned long task1(unsigned long x, unsigned long y) {

                    y < sizeof(unsigned long)*8
    REQUIRES(_____________________________________________________________);
    return x << y;
}
```

5 **Task 2**

```
int task2(int x, int y, int z) {
    REQUIRES(x >= 0 && y >= 0);

                    (y == 0 || x <= INT_MAX/y) && z != 0
    REQUIRES(_____________________________________________________________);
    return (x * y) / z;
}
```

5 **Task 3**

```
int task3(size_t n) {

                    n == 0
    REQUIRES(_____________________________________________________________);

    int x = 0;
    int A[15];
    for (size_t i = 0; i < n; i++) {
        x += A[i];
    }
    return x;
}
```

5 **Task 4**

```
char *task4(size_t n, size_t m) {



            m <= n / sizeof(unsigned int)
            && m <= sizeof(unsigned int)*8
    REQUIRES(____________________________________________________________);

    unsigned int *A = xmalloc(n);
    for (unsigned int i = 0; i < m; i++) {
        A[i] = i << i;
    }
    return A;
}
```

5 **Task 5** For this task, you should assume that casting between signed and unsigned types of the same size works the same way it does in gcc.

```
int task5(signed char n, int m) {




            n >= 0 || m >= 0
    REQUIRES(____________________________________________________________);

    unsigned int x = (unsigned int)(unsigned char)n;
    unsigned int y = (unsigned int)(signed int)n;

    if (x != y) return INT_MIN + m;
    return m;
}
```

5 **Task 6**

```
void task6(int n) {



            n == 1
    REQUIRES(____________________________________________________________);

    int *A = xcalloc(5, sizeof(int));
    for (int i = 0; i < n; i++) {
        free(&A[i]);
    }
}
```

3 High-Speed Data Structures (50 points)

40 **Task 1** Give best and worst-case running time bounds for the following operations. Some of the descriptions state that multiple operations are happening: give the cost of doing the *entire sequence* of operations, not the cost of doing a single operation within the sequence.

Always give the simplest, tightest big-O bound.

	Best-case	Worst-case
Adding n elements to an initially empty queue (implemented as a linked list).	$O(n)$	$O(n)$
Adding n elements to an resizing unbounded array that initially has size 0 and limit 4.	$O(n)$	$O(n)$
Adding n elements to a resizing unbounded array that initially has size 0 and limit $> n$, using **merge-sort** to re-sort the array after **every single addition**.	$O(n^2 \log n)$	$O(n^2 \log n)$
Adding n elements to a resizing unbounded array that initially has size 0 and limit $> n$, using **quicksort** to re-sort the array **only once, after all n additions**.	$O(n \log n)$	$O(n^2)$
Adding a *single* element to a heap data structure that has n elements in it already.	$O(1)$	$O(\log n)$
Adding n elements, which are all or almost all distinct, to an initially empty binary search tree that does not do any re-balancing.	$O(n \log n)$	$O(n^2)$
Adding n elements, which are all or almost all distinct, to an initially empty AVL tree.	$O(n \log n)$	$O(n \log n)$
Inserting n elements, which are all or almost all distinct, to an initially empty non-resizing, separate-chaining hash table with table size m.	$O(n^2/m)$	$O(n^2)$
Looking up a single element in a non-resizing, separate-chaining hash table that already contains n distinct elements and has a table size m.	$O(1)$	$O(n)$
Adding every possible edge to an initially-empty undirected graph with n vertices. Assume an adjacency matrix representation.	$O(n^2)$	$O(n^2)$

10 **Task 2** Grace is implementing a program for the popular word game Scrabble, and she is considering the use of a hash table (using separate chaining) or a balanced binary search tree to store the dictionary of legal words with their definitions. In this case, Grace is not using a separate interface for the data structure. Instead the data structure is being integrated into the full program. (Maybe not such a good idea, but Grace has been programming for many years and wrote very careful data structure invariants.) For the hash table, she would use a hash function that adds up the ASCII values of all of the letters in the word. The binary search tree is ordered by the usual ASCIIbetical ordering.

Which data structure, hash table or binary search tree, would allow her to more easily find all words that start with the letter sequence UNI? Explain your choice in one sentence.

Solution: All the words that start with `UNI` are between `UNI` and `UNJ` in the dictionary, so a binary search tree is going to be the better way of grouping and finding all those words.

Which data structure, hash table or binary search tree, would allow her to more easily find all words that can be formed using the each of letters AERST once? Explain your choice in one sentence.

Solution: For the given hash function, a hash table would put all the words that contain `AERST` once in the same chain, making then easier to find.

4 Generic Exam Question (40 points)

This question involves a slight variant of the C implementation of generic queues: we've removed `queue_size`, `queue_reverse`, and `queue_peek`.

```
typedef bool check_property_fn(void* x);
typedef void* iterate_fn(void* accum, void* x);
typedef void elem_free_fn(void *x);

queue_t queue_new();
  /*@ensures \result != NULL; @*/

bool queue_empty(queue_t Q);
  /*@requires Q != NULL; @*/

void enq(queue_t Q, void *x);
  /*@requires Q != NULL; @*/

void *deq(queue_t Q);
  /*@requires Q != NULL && !queue_empty(Q) > 0; @*/

bool queue_all(queue_t Q, check_property_fn *P);
  /*@requires Q != NULL && P != NULL; @*/

void* queue_iterate(queue_t Q, void *base, iterate_fn *F);
  /*@requires Q != NULL && F != NULL; @*/

void queue_free(queue_t Q, elem_free_fn *F)
  /*@requires Q != NULL; @*/ ;
```

10 **Task 1** Write a C function named `nestring` that matches the type `check_property_fn`. Your function should assume the `void` pointers it is given are either `NULL` or are valid C style strings (type `char*`).

Your function should be written so that `queue_all(Q,&nestring)` will return `false` if any element of the queue is `NULL` or if any element of the queue is a zero-length C-style string. (In other words, `queue_all(Q,&nestring)` should check that everything in the queue is a non-`NULL`, non-empty string.)

Make all casts to and from `void*` explicit.

Solution:

```
bool nestring(void *x) {
   return x != NULL && *(char*)x != '\0';
}
```

15 **Task 2** Recall that if the queue `Q` contains the four elements `e1`, `e2`, `e3`, and `e4`, then calling `queue_iterate(Q,base,&f)` will compute

```
f(f(f(f(base,e1),e2),e3),e4)
```

whereas if `Q` is empty `queue_iterate(Q,base,&f)` will just return `base`.

Respecting the interface of queues above, write `queue_size` that takes a queue and returns an integer representing the number of elements in the queue. Your solution must use `queue_iterate` (not `deq` or `enq`), and you'll need to define a helper function.

For full credit, don't heap-allocate any memory with (`x`)`malloc` or (`x`)`calloc`.

Make all casts to and from `void*` explicit.

Solution:

```
void *counter(void *accum, void *x) {
  REQUIRES(accum != NULL);
  *(int*)accum += 1;
  return accum;
}

int queue_size(queue_t Q) {
  REQUIRES(Q != NULL);
  int i = 0;
  queue_iterate(Q, (void*)&i, &counter);
  return i;
}
```

(Also, don't cast between integer types like `int` and pointer types like `void*`. This isn't something we even mentioned the possibility of during class, so you don't know what this means you probably are not going to do it.)

15 **Task 3** Here are two different *implementations* of `queue_all`:

```
/* Implementation A */
bool queue_all(queue *Q; check_property_fn *P) {
  REQUIRES(is_queue(Q) && P != NULL);
  for (list *L = Q->front; L != NULL; L = L->next)
    if (!(*P)(L->data)) return false;
  return true;
}
/* Implementation B */
bool queue_all(queue *Q; check_property_fn *P) {
  REQUIRES(is_queue(Q) && P != NULL);
  bool res = true;
  for (list *L = Q->front; L != NULL; L = L->next)
    res = (*P)(L->data) && res;
  return res;
}
```

Write a `main` function (and any necessary helper functions) that respects the queue interface but that returns 0 if the queue is implemented with implementation A and that returns 1 if the queue is implemented with implementation B. You can allocate memory freely and ignore memory leaks.

Solution: High level explanation: the `bool` returned by both impelementations is *always* the same, but `A` stops as soon as we get a `false` and `B` runs the function pointer on every data element.

So the way to distinguish the two implementations is to have the property checker modify the queue, and see if the whole queue gets modified, or just the whole queue up until the first element that causes the property checker to return `false`.

```
bool tester(void *x) {
  REQUIRES(x != NULL);
  *(int*)x += 10;
  return false;
}

int main() {
  queue_t Q = queue_new();
  enq(Q, xcalloc(1, sizeof(int)));
  enq(Q, xcalloc(1, sizeof(int)));
  queue_all(Q, &tester);
  deq(Q);
  if (*(int*)deq(Q) == 10) return 1;
  return 0;
}
```

Hint: it may be necessary for your the function you pass `queue_all` to modify memory.

If you're stumped: for partial (half) credit, you can explain the difference between the two functions without writing a test case that differentiates them.

5 Graph Expansion (30 points)

In this question, we use the following modified interface of undirected graphs, which incorporates ideas from unbounded arrays.

```
typedef unsigned int vertex;

unsigned int graph_size(graph_t G);
  //@requires G != NULL;

graph_t graph_new();
  //@ensures \result != NULL'
  //@ensures graph_size(\result) == 0;

void graph_grow(graph_t G);
  //@requires G != NULL;

bool graph_hasedge(graph_t G, vertex v, vertex w);
  //@requires G != NULL;
  //@requires v < graph_size(G) && w < graph_size(G);

void graph_addedge(graph_t G, vertex v, vertex w);
  //@requires G != NULL;
  //@requires v < graph_size(G) && w < graph_size(G);
  //@requires v != w && !graph_hasedge(G, v, w);

void graph_free(graph_t G);
  //@requires G != NULL;
```

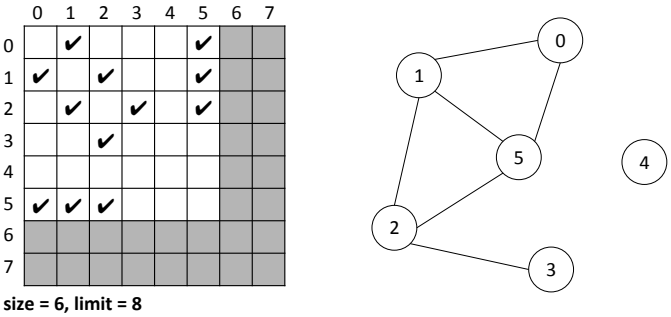
A new graph is always created with 0 vertices (meaning `graph_size` is initially 0), and `graph_grow` increments `graph_size` by one:

```
graph_t G = graph_new();
graph_grow(G); // Adds the vertex 0
graph_grow(G); // Adds the vertex 1
graph_addedge(G, 0, 1);
graph_grow(G); // Adds the vertex 2
graph_grow(G); // Adds the vertex 3
graph_addedge(G, 2, 3);
graph_addedge(G, 0, 3);
assert(graph_size(G) == 4); // 4 vertices, numbered 0, 1, 2, and 3
```

5 **Task 1** We create a large graph, and in the process call `graph_new` once, call `graph_grow` k times, and call `graph_addedge` $3k$ times. Is the graph sparse or dense?

Solution: Sparse: the number of edges would need to be proportional to k^2 for it to be dense.

We can implement this interface using adjacency matrices: we'll hold the adjacency matrix in a 1-D array, using the same layout we used for the images assignment, where the element in row `i` and column `j` is stored at index `i*G->limit + j` in the array.




```
typedef struct graph_header graph;
struct graph_header {
    size_t size;
    size_t limit;
    bool *adj;
};
```

A well-formed undirected graph (the `is_graph(G)` data structure invariant) is a non-`NULL` struct with `size` strictly less than `limit`, a non-`NULL` array `adj` with length `limit*limit`. In C, it is impossible to actually check this last condition.

Because the graphs are undirected, we also require that `G->adj[i*G->limit + j]` is equal to `G->adj[j*G->limit + i]` for every `i` and `j` in the range `[0..G->limit)`.

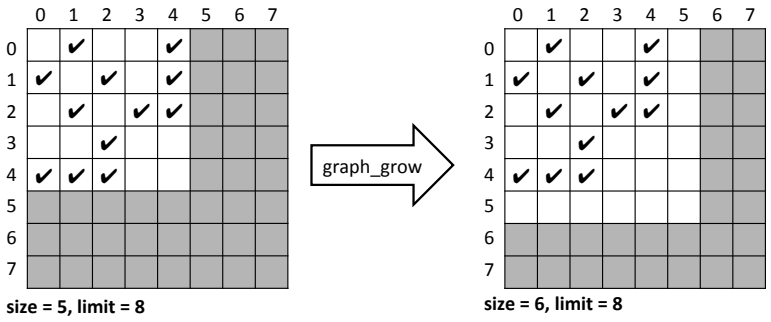
5 **Task 2** Implement `graph_addege` for the data structure described above.

```
void graph_addedge(graph *G, vertex v, vertex w) {
    REQUIRES(is_graph(G) && v < G->size && w < G->size);
    REQUIRES(v != w && !graph_hasedge(G, v, w));

    G->adj[i*G->limit + j] = true;
    G->adj[j*G->limit + i] = true;

    ENSURES(is_graph(G));
}
```

When we call `graph_grow`, we increase the `size`: if it's still less than `limit`, we're done.



Otherwise, we double `limit`, which quadruples the length of the array.

10 **Task 3** Implement `graph_grow` according to the description above. Avoid memory leaks.

```
void graph_grow(graph *G) {
    REQUIRES(is_graph(G));
    G->size += 1;
    if (G->size == G->limit) {
        assert(limit*limit < SIZE_T_MAX / 4); // Not required
        bool *OLD = G->adj;
        bool *NEW = xcalloc(limit*limit*4, sizeof(bool));
        for (size_t row = 0; row < limit; row++) {
           for (size_t col = 0; col < limit; col++) {
              NEW[row*limit*2 + col] = OLD[row*limit + col];
           }
        }
        G->limit *= 2;
        G->adj = NEW;
        free(OLD);
    }
    ENSURES(is_graph(G));
}
```

10 **Task 4** In most cases, `graph_grow` requires 0 array writes. In the worst case, we will have `size` $=$ `limit` $= n$, and the running time of `graph_grow` in terms of n will be in

$O(\, n^2 \,)$

Assuming the array has resized before, that expensive operation was necessarily preceded by exactly...

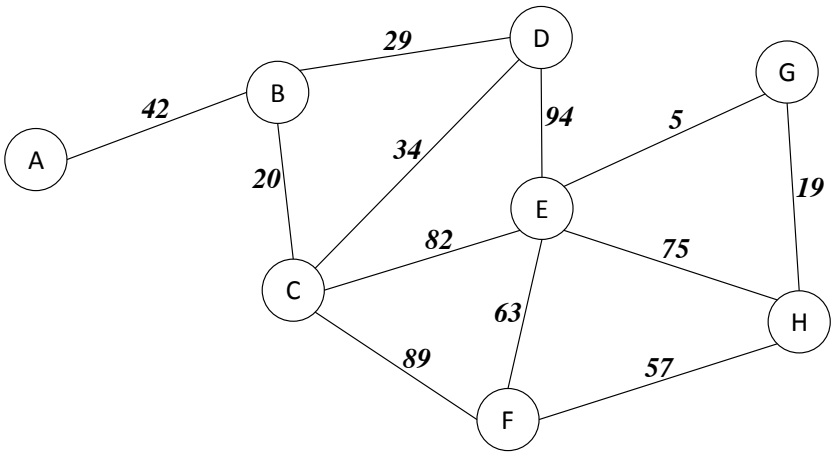
$n/2 - 1$

...cheap calls to `graph_grow`. That means that the amortized cost of `graph_grow` can be said to be in

$O(\, n \,)$

6 Minimum Spanning Trees (20 points)

We can apply Kruskal's algorithm to find a minimum spanning tree for the graph shown below:



In the table below, fill in the edges in the order considered by Kruskal's algorithm and indicate for each whether it would be added to the spanning tree (**Yes**) or not (**No**). Do not list edges that would not even be considered. We have filled in the first two edges for you.

EDGE CONSIDERED:	ADDED TO MST?
(E,G)	Yes
(G,H)	Yes
(B,C)	Yes
(B,D)	Yes
(C,D)	No
(A,B)	Yes
(F,H)	Yes
(E,F)	No
(E,H)	No
(C,E)	Yes

(C,F) and (D,E) not considered