16-311-Q Introduction to Robotics Fall'17

Lecture 24:
Data Association
Line Features

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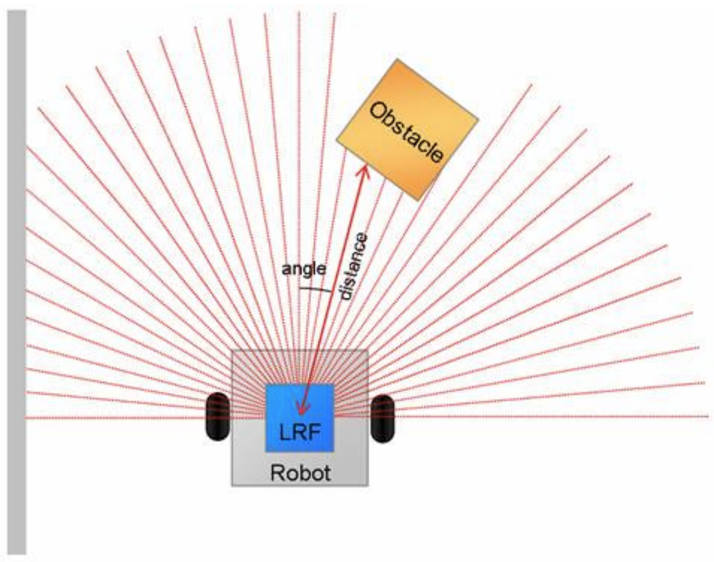
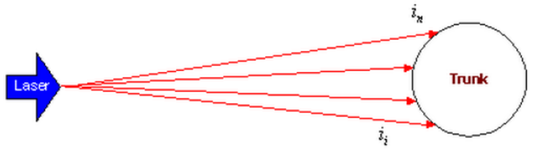
FEATURE EXTRACTION FROM RANGE DATA

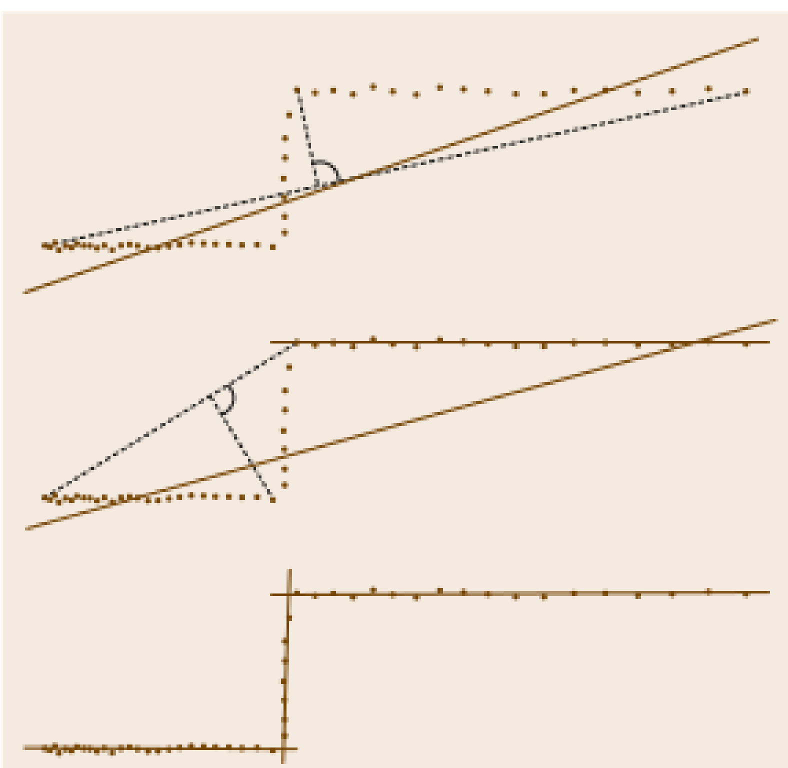


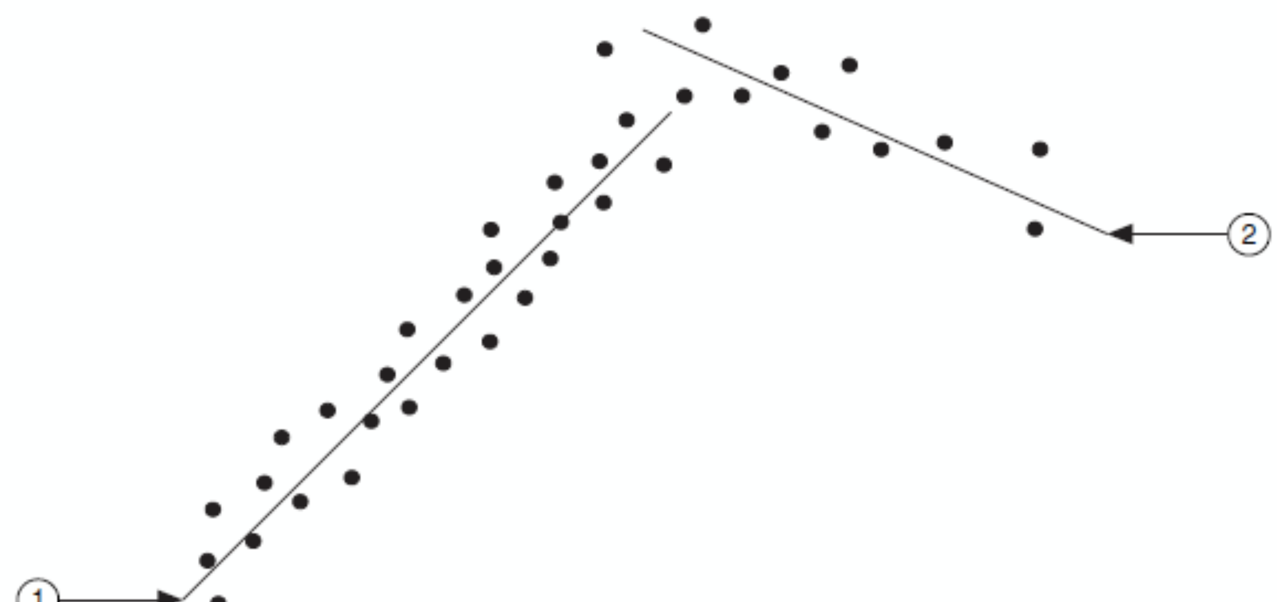


- Commonly, map features extracted from ranging sensors are geometric primitives:
  - <u>Line segments</u>
  - <u>Circles</u>
  - Ellipsis
  - Regular polygons
- These geometric primitives can be expressed in a compact parametric form and enjoys closed-form solutions
- Let's focus on <u>line segments</u>, the simplest (yet very useful) features to extract

CHALLENGES IN LINE EXTRACTION

1. How many lines are there?
2. Which points belong to which line?
3. Given the points that belong to a line, how to estimate the line model parameters (accounting for sensing uncertainties)?



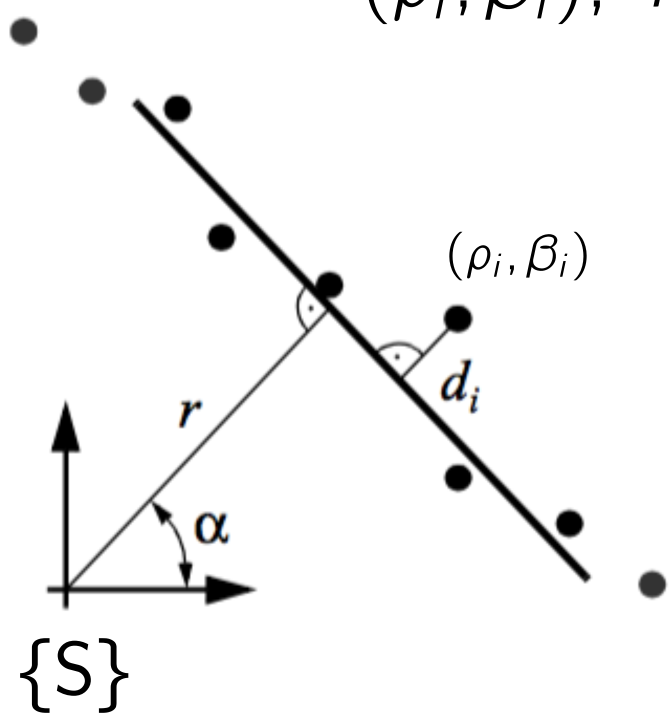


Let's start by looking at problem 3...

PROBABILISTIC LINE FITTING

- **Scenario:** Using a range sensor, the robot gathers n measurement points in polar coordinates in the robot's sensor frame {S}

$$(\rho_i, \beta_i), \quad i = 1, \ldots, n$$



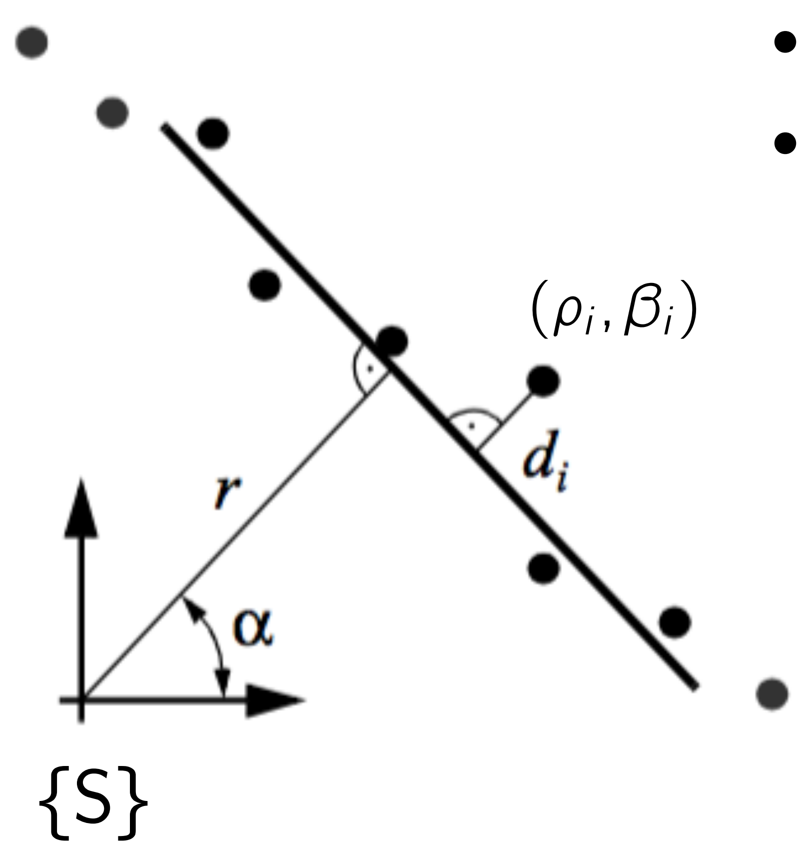
- Because of sensor noise each measurement in range and bearing is modeled as a bivariate Gaussian random variable: $X_i = (R_i,\ B_i)$

$$R_i \sim N(\rho_i, \sigma^2_{\rho_i}), \quad B_i \sim N(\beta_i, \sigma^2_{\beta_i})$$

- $R_i,\ B_i$ are considered as independent Gaussian variables

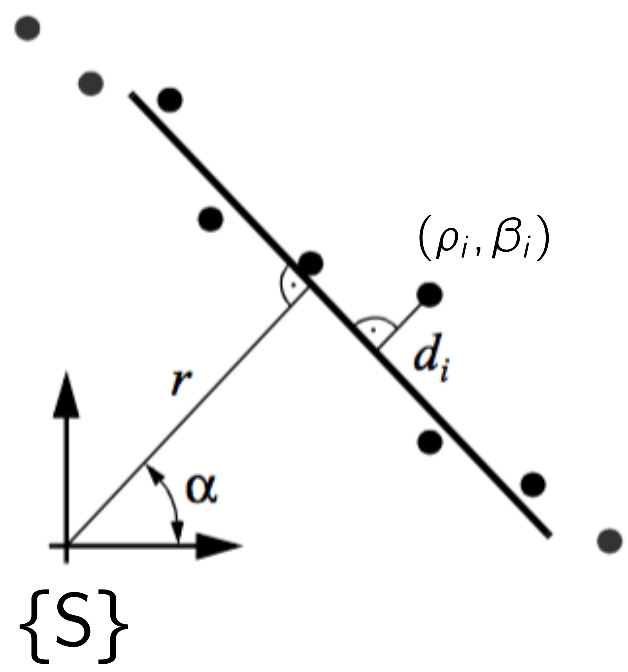
- Given a measurement point (ρ, β), the corresponding Cartesian coordinates in {S} are: x=ρ cosβ, y=ρ sinβ
- **If there were no error:** all points would lie on a unique line that would be described in polar coordinates by its distance r and orientation $\boldsymbol{\alpha}$ with respect to {S}:

$$\rho \cos \beta \cos \alpha + \rho \sin \beta \sin \alpha - r = \rho \cos(\beta - \alpha) - r = 0$$



- Unfortunately, there are errors!
- For a measurement i, the error in the sense of the orthogonal distance from the line:

$$\rho_i \cos(\beta_i - \alpha) - r = d_i$$



Sum of (unweighted) squared errors:

$$S = \sum_i d_i^2 = \sum_i (\rho_i \cos(\beta_i - \alpha) - r)^2$$

Line parameters that minimize S:

$$\frac{\partial S}{\partial \alpha} = 0, \qquad \frac{\partial S}{\partial r} = 0$$

Sum of squared errors weighted by measure uncertainty:

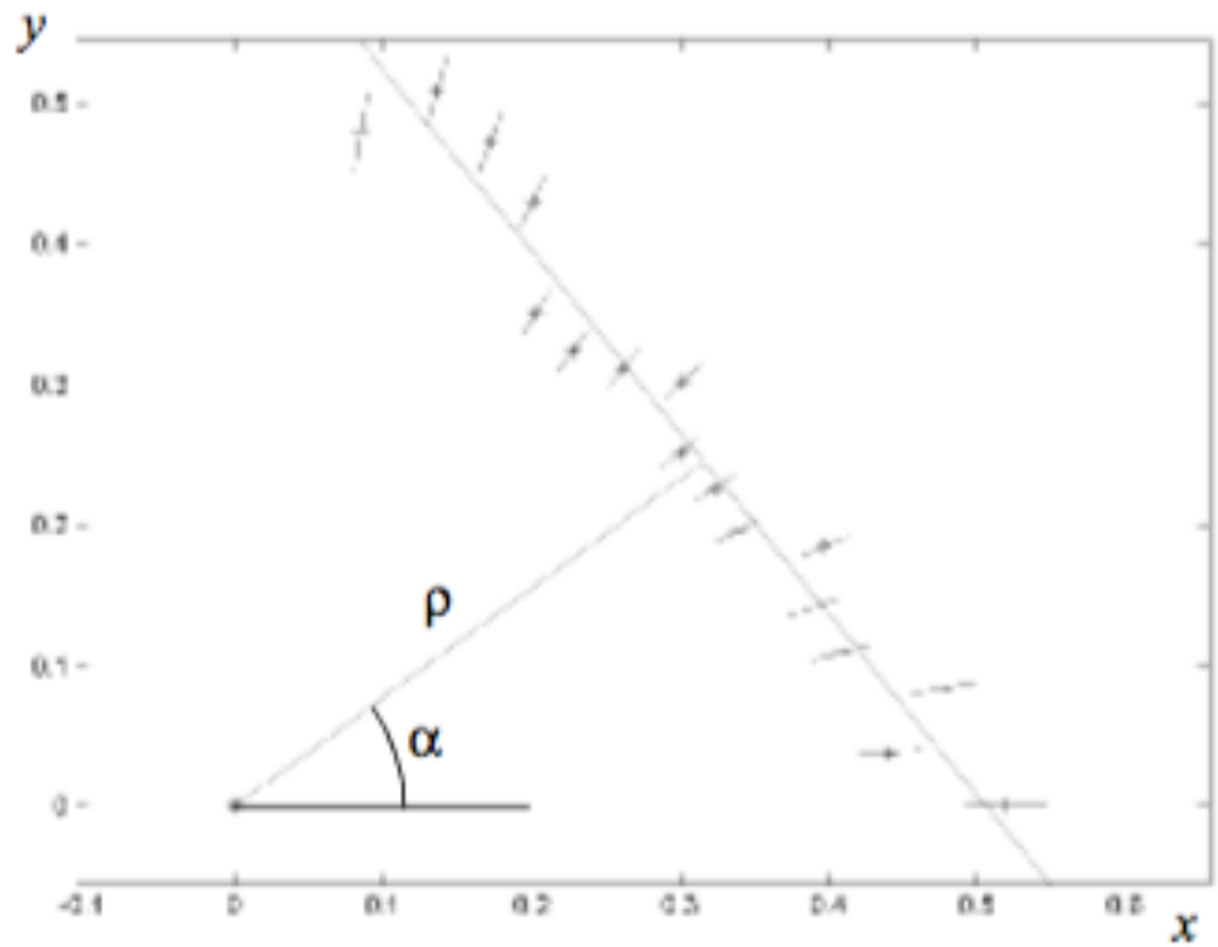
$$w_i = 1/\sigma_{\rho_i}^2 \quad \rightarrow \quad S = \sum_i w_i d_i^2 = \sum_i w_i (\rho_i \cos(\beta_i - \alpha) - r)^2$$

$$\alpha = \frac{1}{2}\operatorname{atan}\left(\frac{\sum_i w_i \rho_i^2 \sin 2\beta_i - \frac{2}{\sum w_i} \sum \sum w_i w_j \rho_i \rho_j \cos\beta_i \sin\beta_j}{\sum_i w_i \rho_i^2 \cos 2\beta_i - \frac{1}{\sum w_i} \sum \sum w_i w_j \rho_i \rho_j \cos(\beta_i + \beta_j)} \right)$$

$$r = \frac{\sum_i w_i \rho_i \cos(\beta_i - \alpha)}{\sum w_i}$$

NUMERIC EXAMPLE

pointing angle of sensor θ_i [deg]	range ρ_i [m]
0	0.5197
5	0.4404
10	0.4850
15	0.4222
20	0.4132
25	0.4371
30	0.3912
35	0.3949
40	0.3919
45	0.4276
50	0.4075
55	0.3956
60	0.4053
65	0.4752
70	0.5032
75	0.5273
80	0.4879



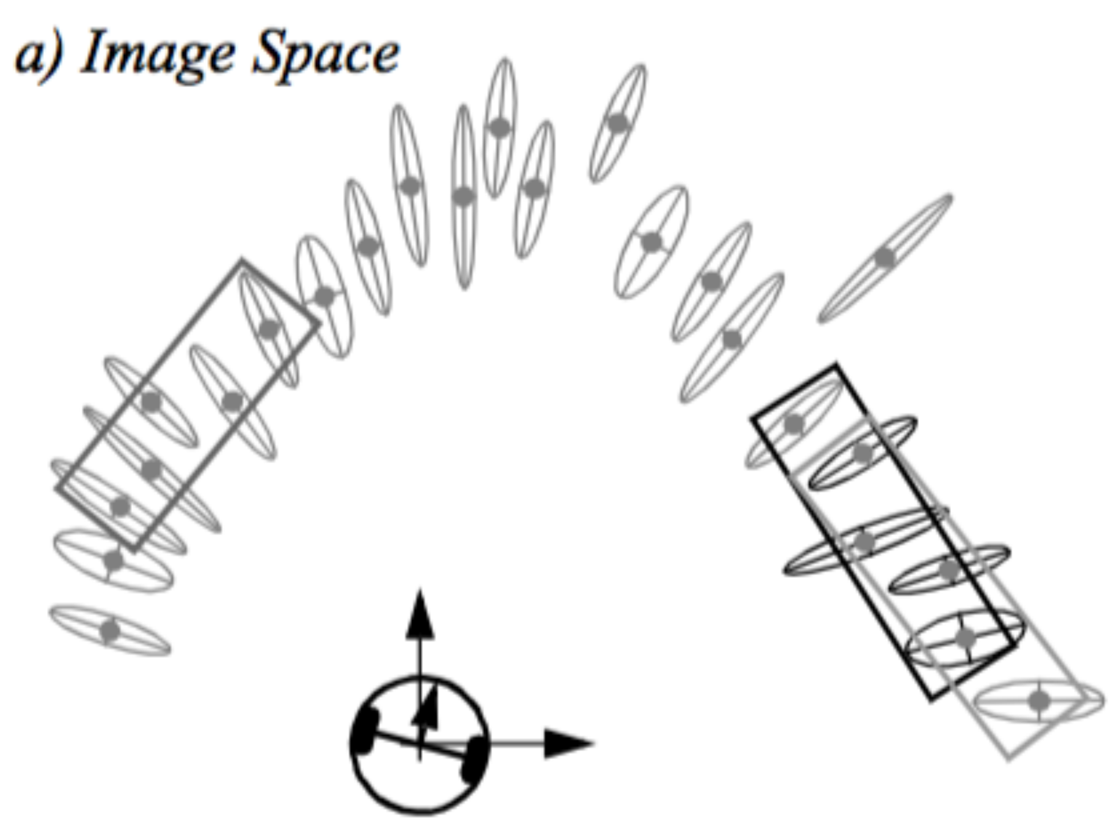
$\boldsymbol{\alpha} = 37.36$

$r = 0.4$

Uncertainty proportional to the distance

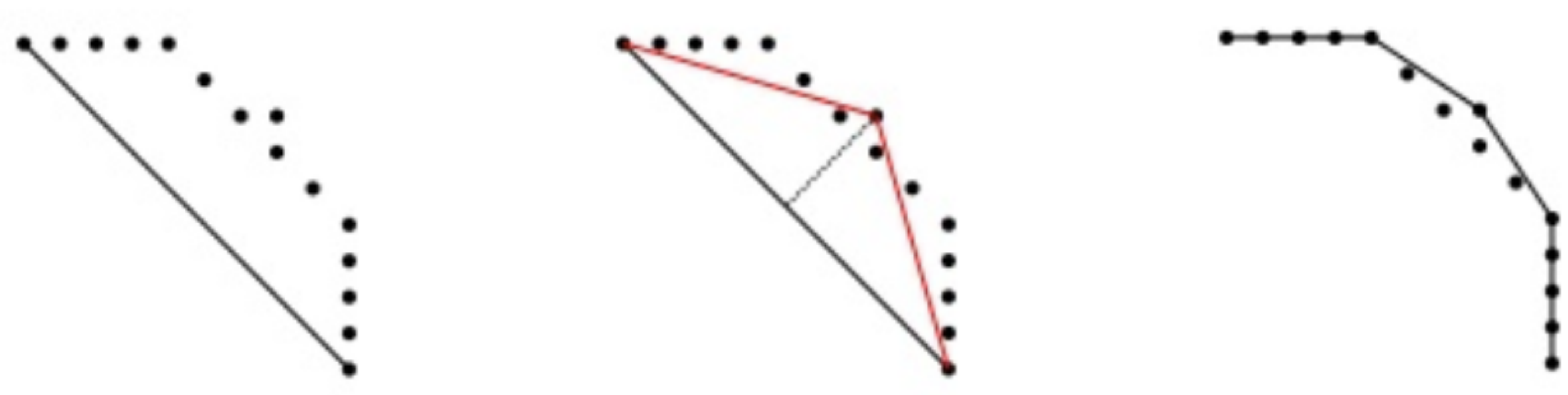
SEGMENTATION

- Which measurements points are part of a line?
- **Segmentation**: Dividing up a set of measurements into subsets, necessary for line extraction



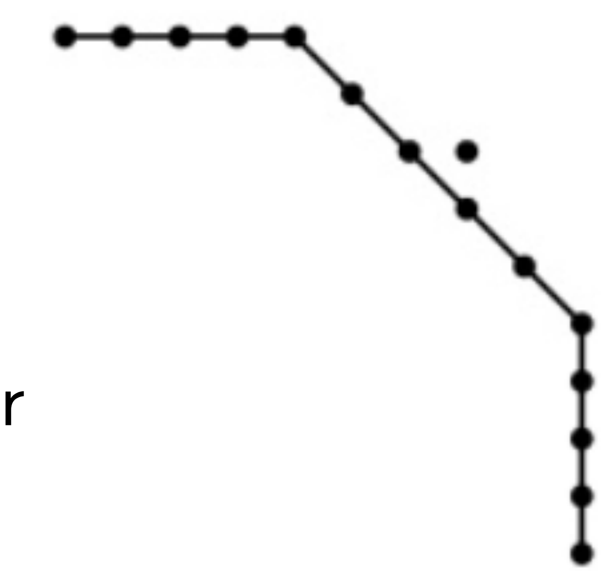
A set of n_f neighboring points of the image space

SPLIT-AND-MERGE



Keep slitting until a distance to a line fit is greater than a threshold

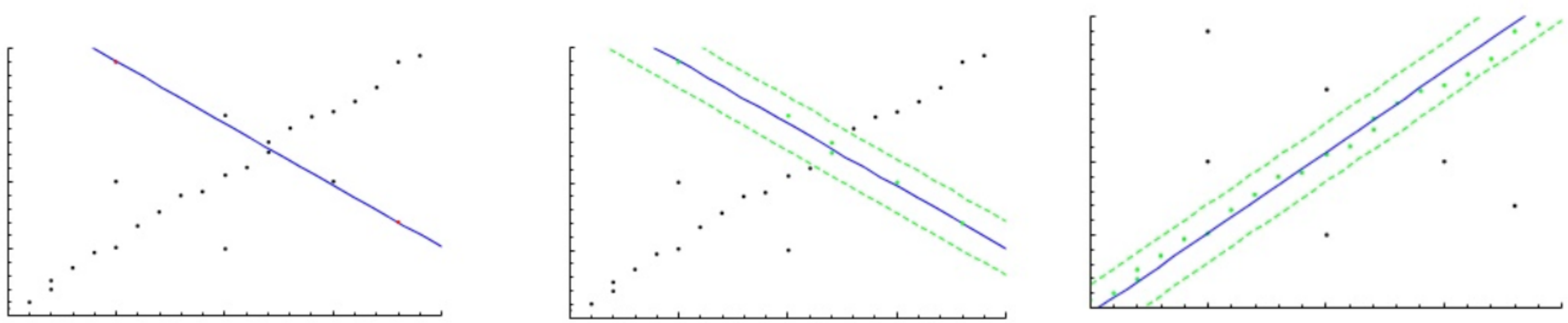
If only one point in a subset, it's treated as an outlier



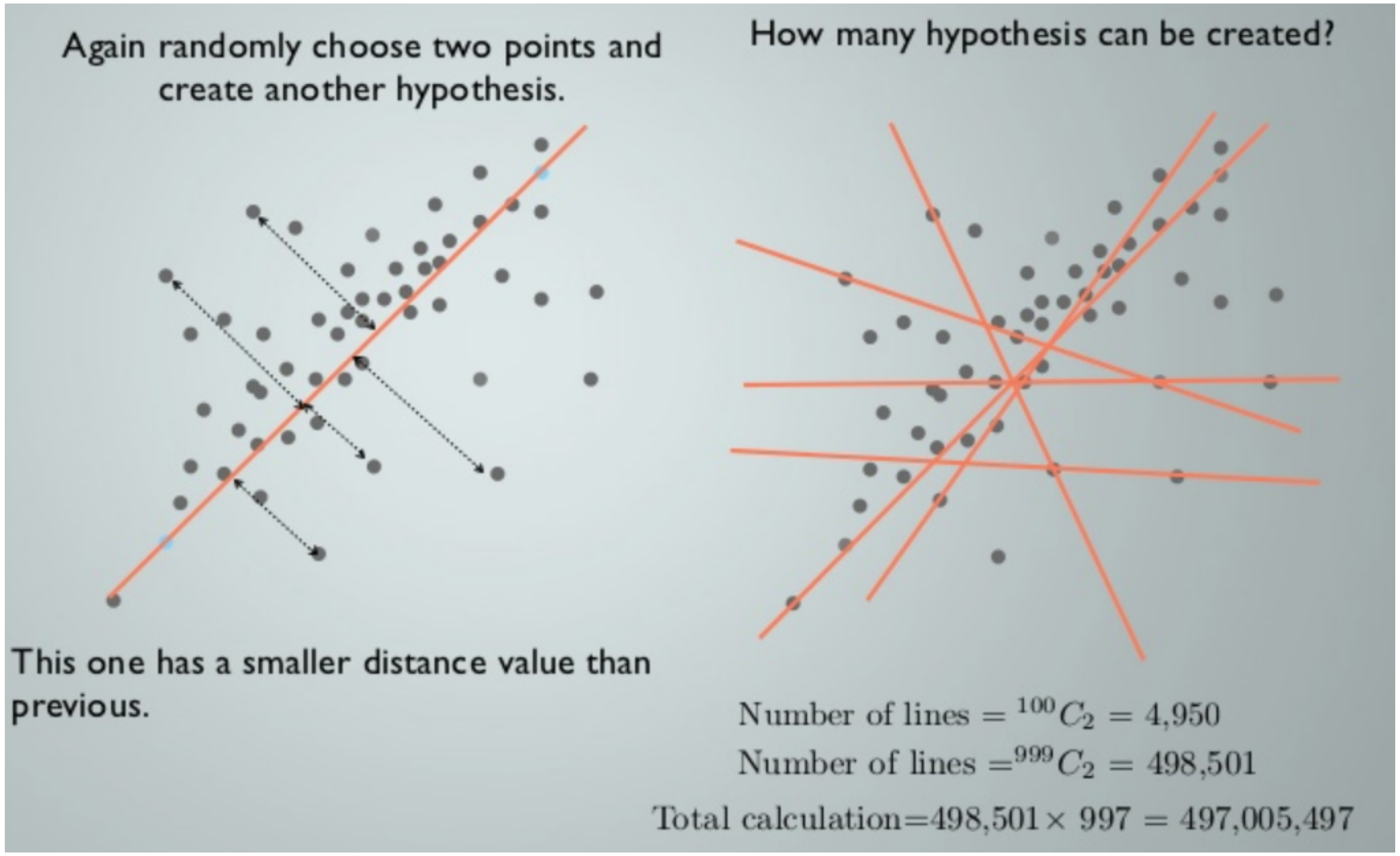
RANSAC

Random Sample Consensus: data fitting robust to outliers

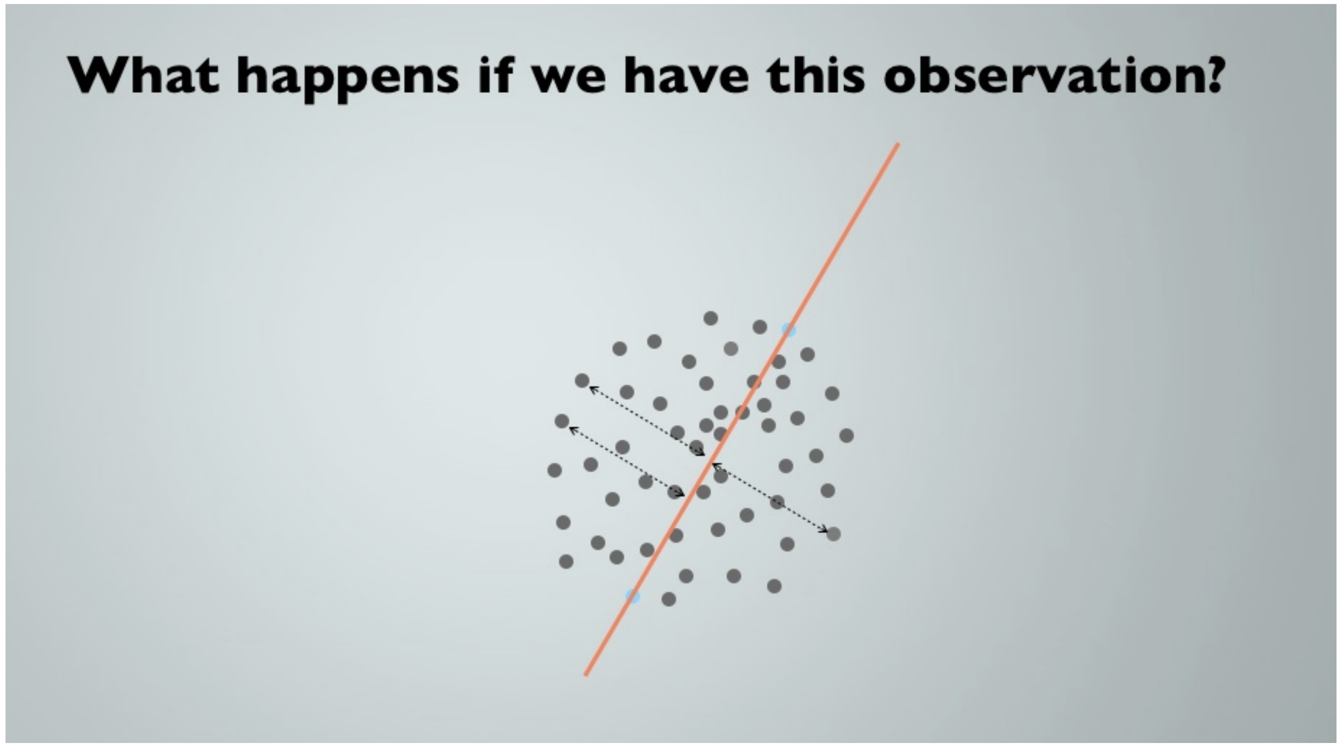
1. Initial: let A be a set of N points
2. **repeat**
3. Randomly select a sample of 2 points from A
4. Fit a line through the two points
5. Compute the distance to all the others points to the line
6. Construct the inliner set: count the number of points with distance to the line $< d$
7. Store the inliers
8. **until** Max number of iterations k
9. *Return the set with the largest number of inliers*

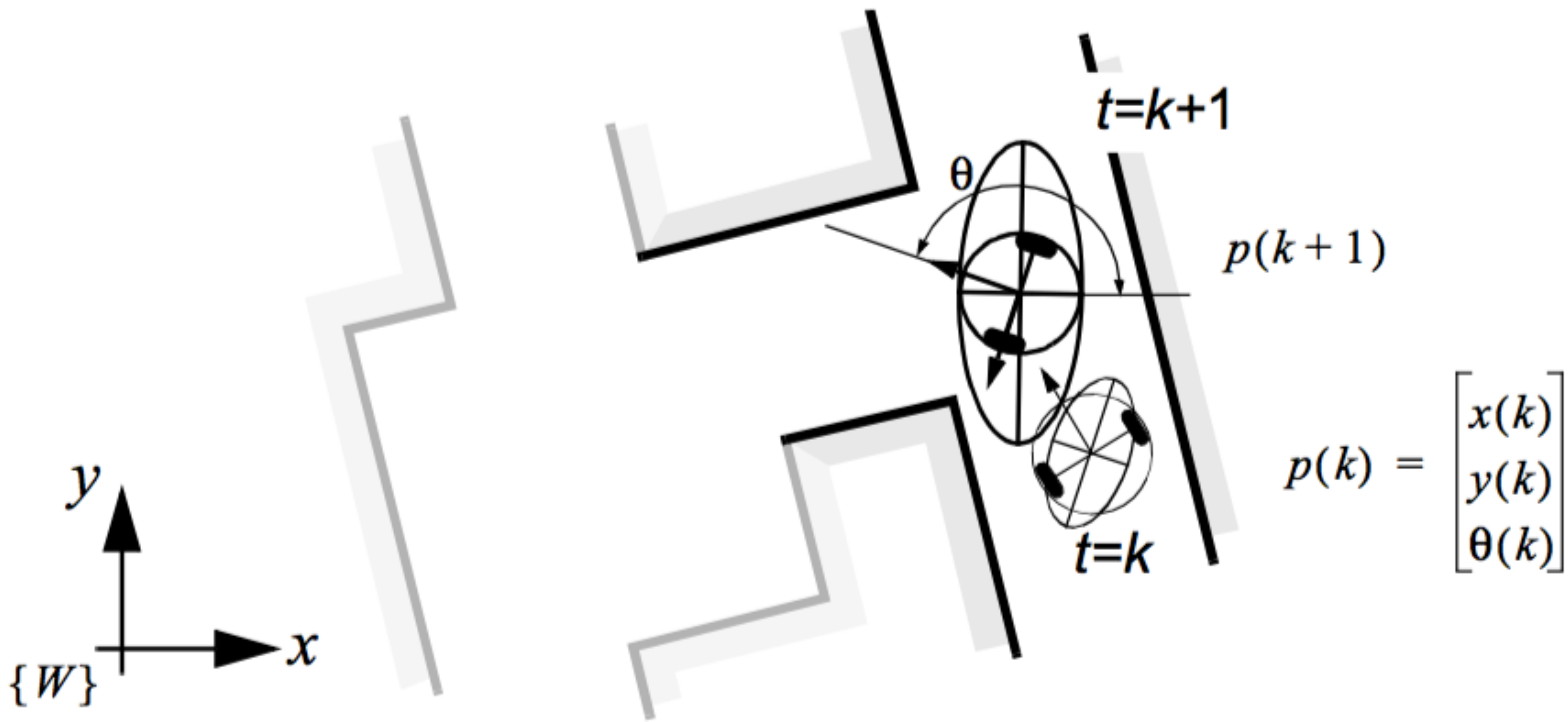


SAMPLING POINTS



Extract multiple lines: iteratively remove the found line points





t=k+1
θ
p(k + 1)
p(k) = [x(k) y(k) θ(k)]
t=k
y
x
{W}

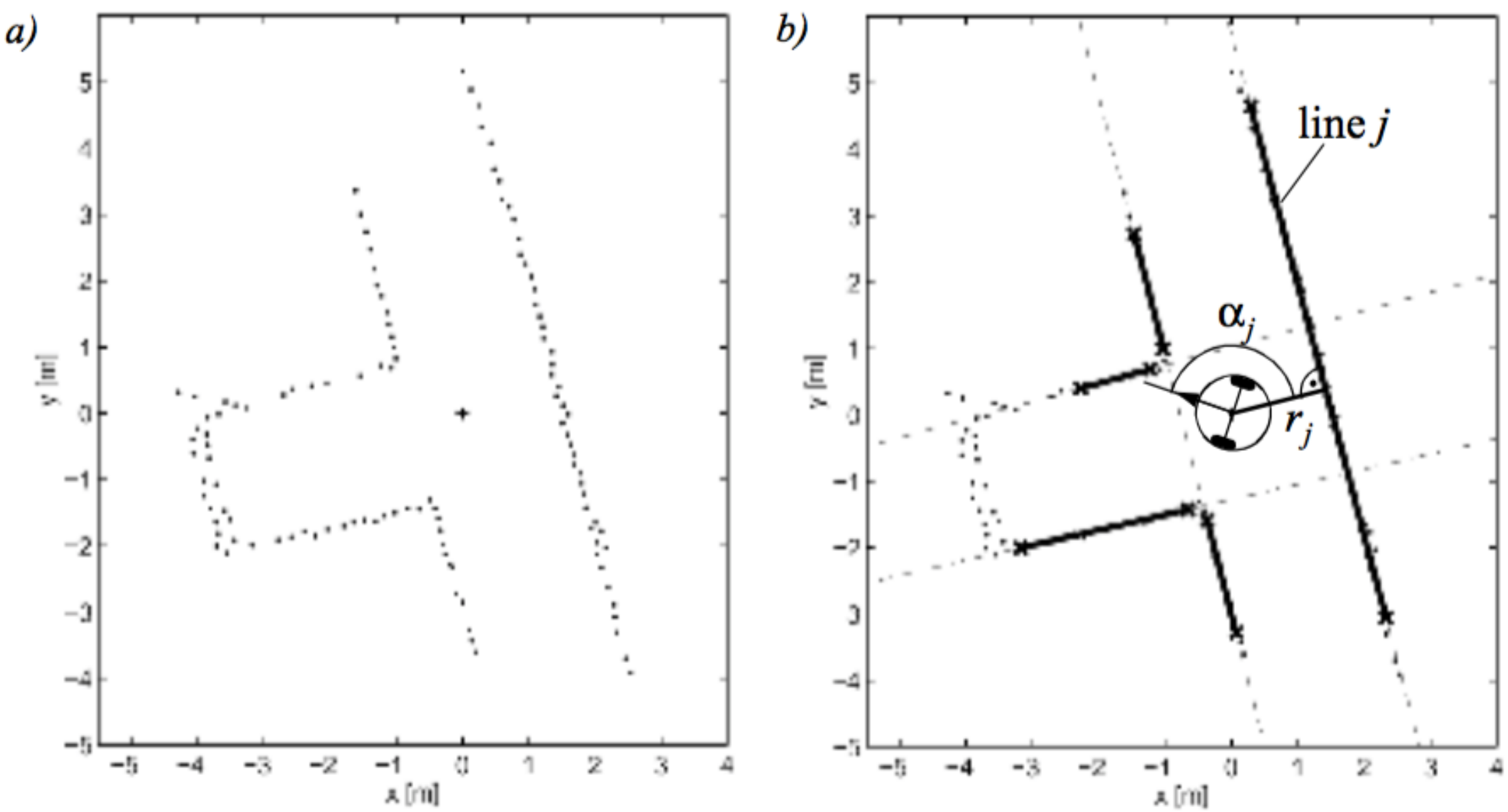
OBSERVATIONS = EXTRACTED LINE FEATURES

2. Observation. For line-based localization, each single observation (i.e., a line feature) is extracted from the raw laser rangefinder data and consists of the two line parameters $\beta_{0,j}$, $\beta_{1,j}$ or α_j, r_j (figure 4.36) respectively. For a rotating laser rangefinder, a representation in the polar coordinate frame is more appropriate and so we use this coordinate frame here:

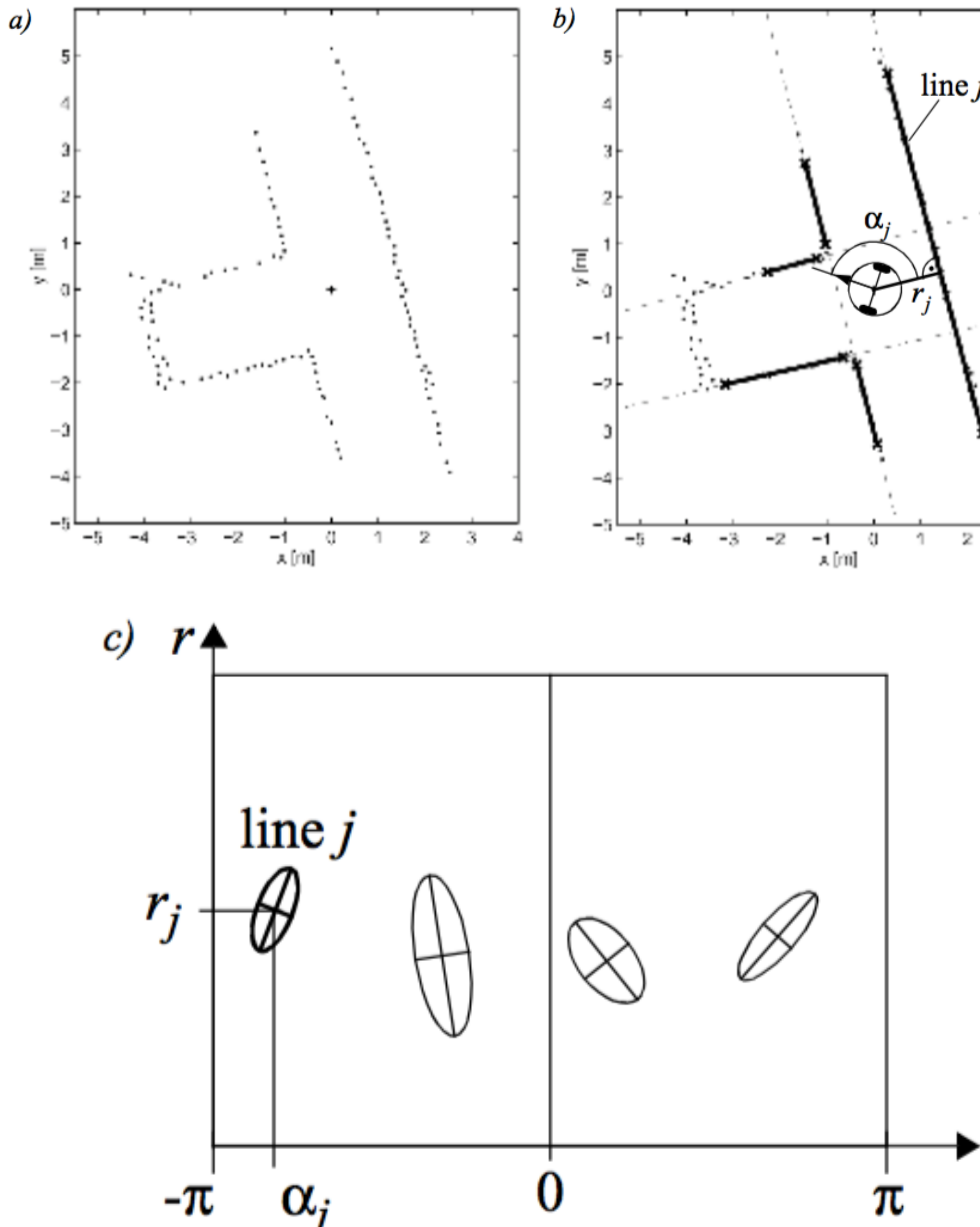
$$z_j(k+1) = {}^R\begin{bmatrix} \alpha_j \\ r_j \end{bmatrix}$$

Parameters describing each line found

$$\Sigma_{R,j} = \begin{bmatrix} \sigma_{\alpha\alpha} & \sigma_{\alpha r} \\ \sigma_{r\alpha} & \sigma_{rr} \end{bmatrix}_j$$



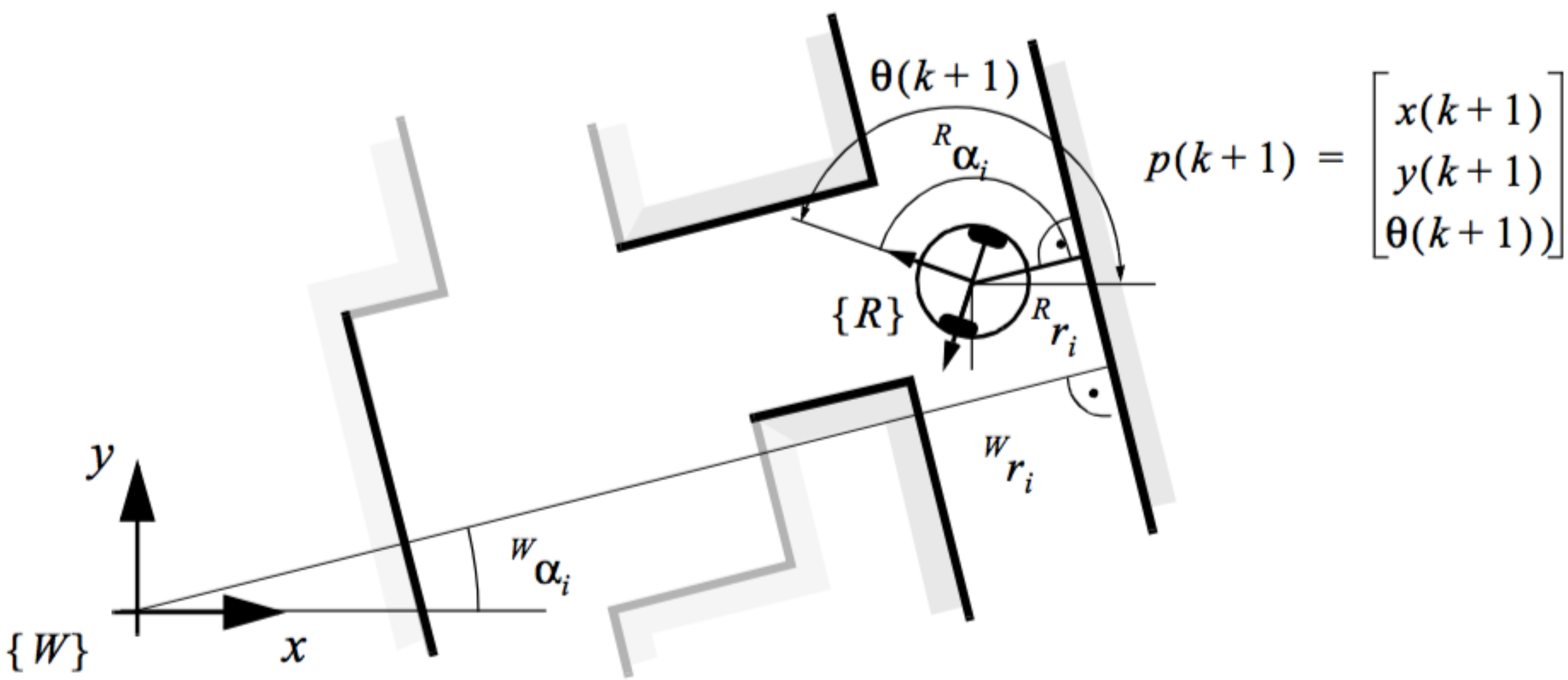
SENSOR VS. MODEL SPACE



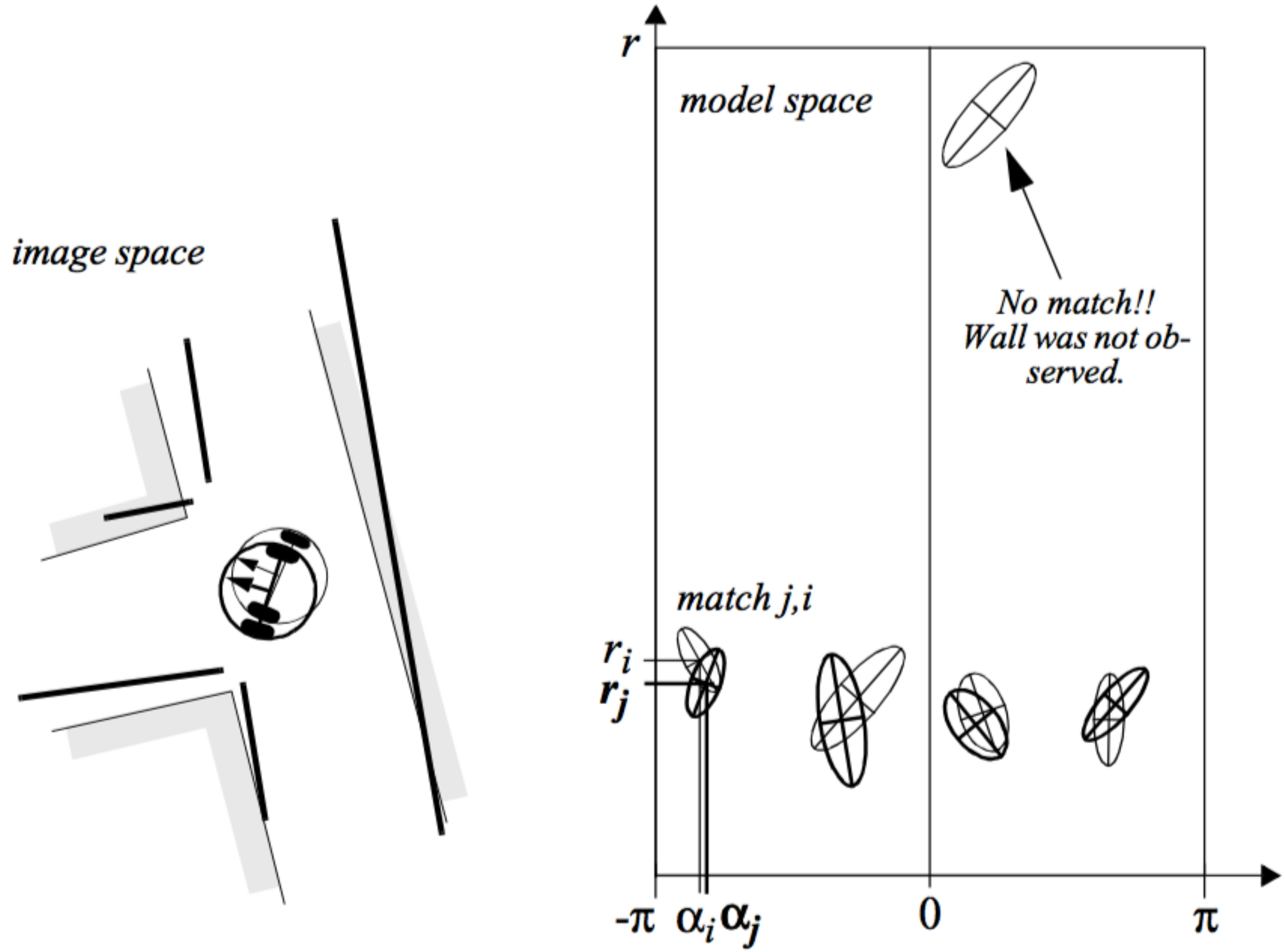
FROM MAP TO SENSOR/ROBOT SPACE

$$\hat{z}_i(k+1) = {}^R\begin{bmatrix} \alpha_{t,i} \\ r_{t,i} \end{bmatrix} = h_i(z_{t,i}, \hat{p}(k+1|k))$$

$$= \begin{bmatrix} {}^W\alpha_{t,i} - {}^W\hat{\theta}(k+1|k) \\ {}^Wr_{t,i} - ({}^W\hat{x}(k+1|k)\cos({}^W\alpha_{t,i}) + {}^W\hat{y}(k+1|k)\sin({}^W\alpha_{t,i})) \end{bmatrix}$$



DATA ASSOCIATION USING VALIDATION GATES



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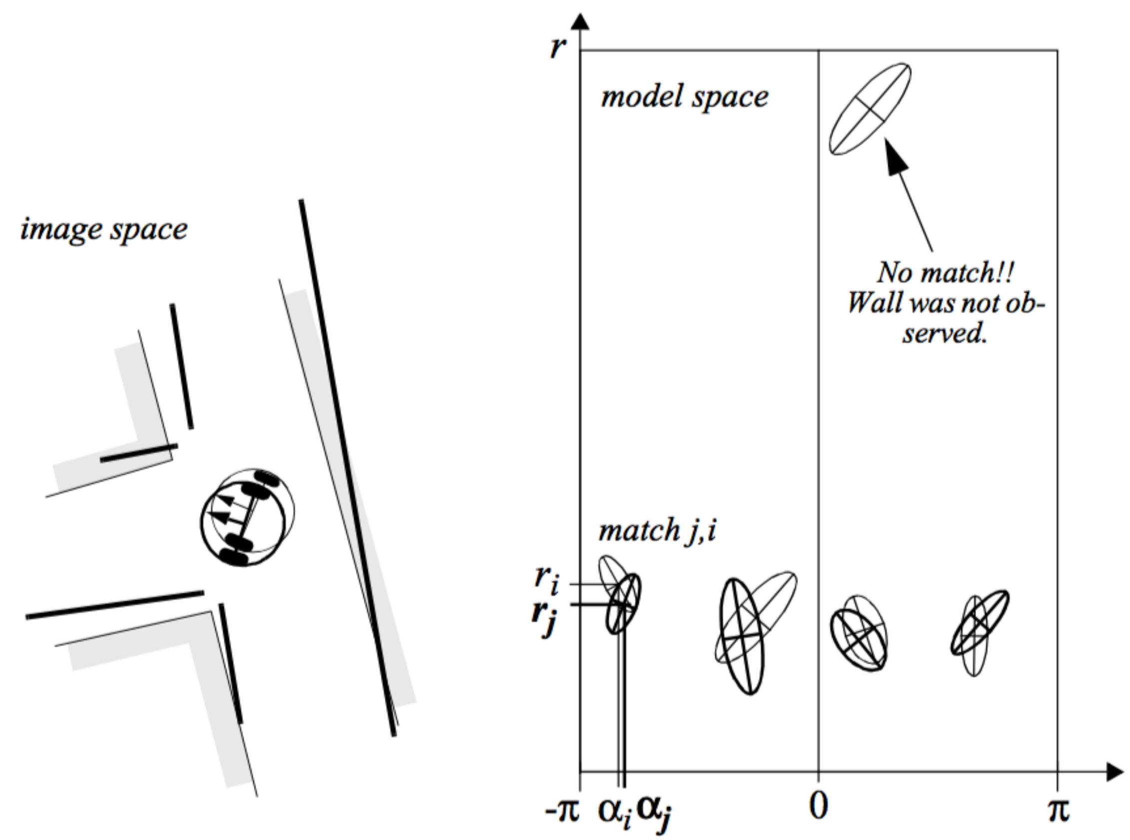

$$v_{ij}^T(k+1) \cdot \Sigma_{IN,\,ij}^{-1}(k+1) \cdot v_{ij}(k+1) \le g^2$$

with

$$\begin{aligned} v_{ij}(k+1) &= [z_j(k+1) - h_i(z_t, \hat{p}(k+1|k))] \\ &= \begin{bmatrix} \alpha_j \\ r_j \end{bmatrix} - \begin{bmatrix} {}^W\alpha_{t,\,i} - {}^W\hat{\theta}(k+1|k) \\ {}^Wr_{t,\,i} - ({}^W\hat{x}(k+1|k)\cos({}^W\alpha_{t,\,i}) + {}^W\hat{y}(k+1|k)\sin({}^W\alpha_{t,\,i})) \end{bmatrix} \end{aligned}$$

$$\Sigma_{IN,\,ij}(k+1) = \nabla h_i \cdot \Sigma_p(k+1|k) \cdot \nabla h_i^T + \Sigma_{R,\,i}(k+1)$$

UPDATED POSE AND MAP