16-311-Q Introduction to Robotics Fall'17

Lecture 25:
Bayesian Filters
Monte Carlo Localization (PF)

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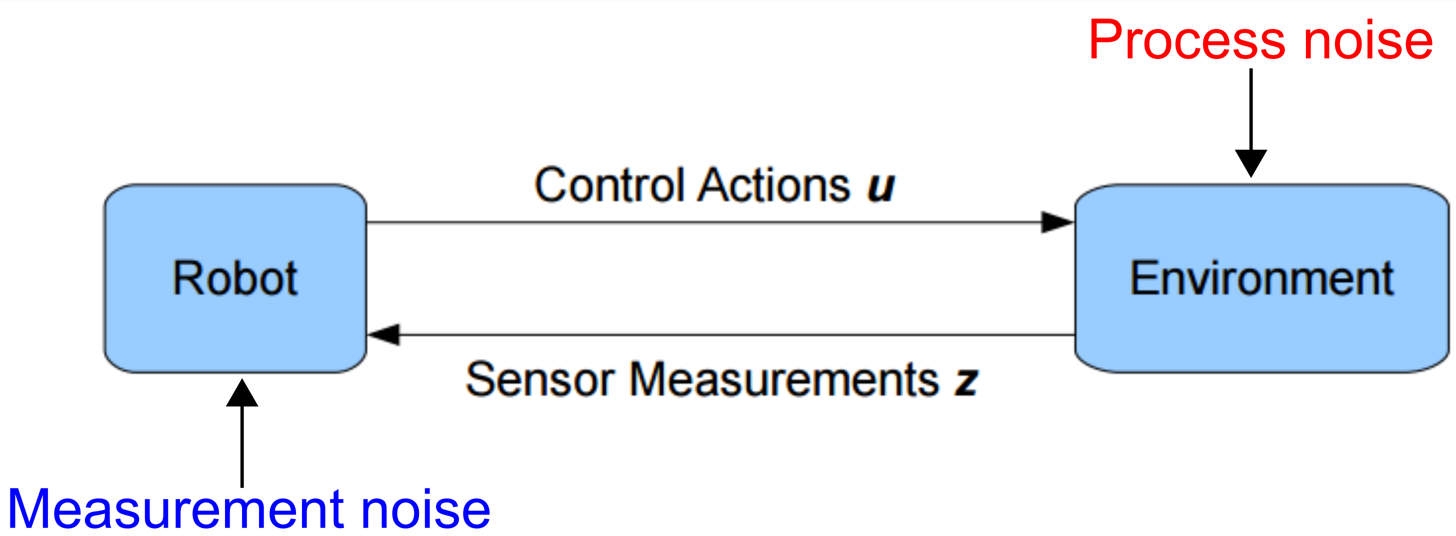
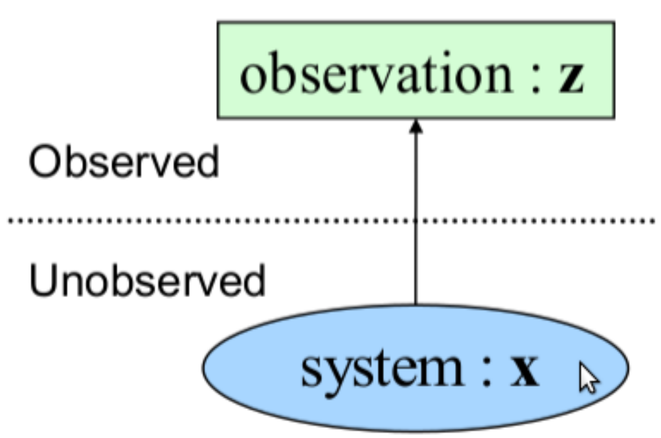
PROBABILISTIC INFERENCE



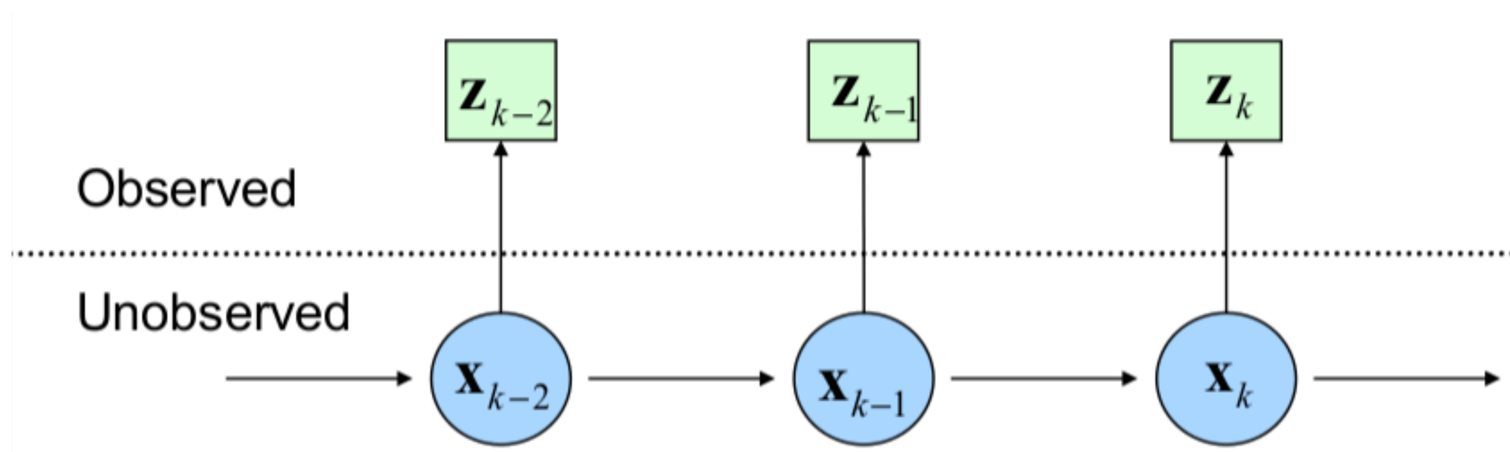
Localization is an instance of the more general problem of state estimation in a noisy (feedback-based) controlled system

- ***Probabilistic inference*** is the problem of estimating the *hidden variables* (states or parameters) of a system in an optimal and consistent fashion (using probability theory), given noisy or incomplete observations.



Given z, what can we infer about x?

For a robot typically the system evolves over time $\rightarrow$ Sequential probabilistic inference: Estimate $\boldsymbol{x}_k$ given $\boldsymbol{z}_{1:k}$ and information about system's dynamics and about how observations are obtained



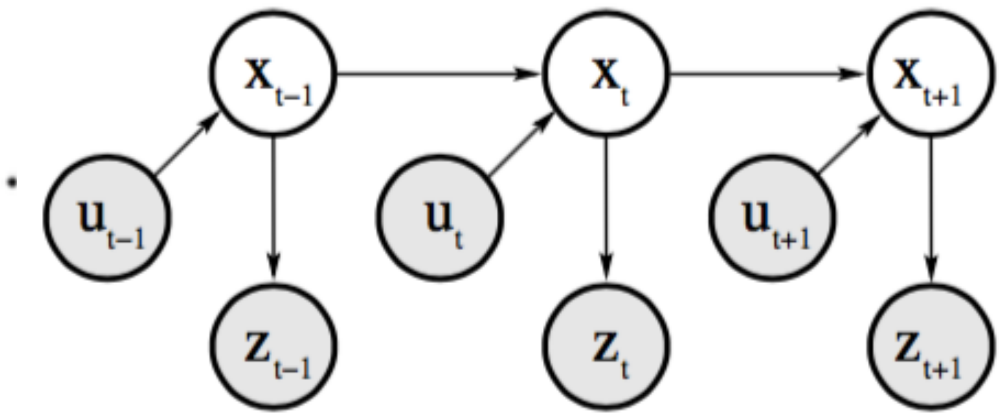
PROBABILISTIC INFERENCE

- Initial state $\boldsymbol{x}_0$, or prior probability distribution of the system state: $p(\boldsymbol{x}_0)$
- Stream of observations $\boldsymbol{z}$ and action data $\boldsymbol{u}$: $\{u_1, z_1, u_2, z_2, \ldots, u_t, z_t\}$
- State transition dynamics = state transition probability distribution: $p(\boldsymbol{x}_t \mid \boldsymbol{x}_{0:t-1}, \boldsymbol{u}_{1:t})$
  $\rightarrow$ Action Model: How the state change over time in relation to the given control actions and *disturbances*
- Measurement probability: $p(\boldsymbol{z}_t \mid \boldsymbol{x}_{0:t})$
  $\rightarrow$ Sensor Model: How probable is measurement $\boldsymbol{z}_t$ given the assumed current state
- Posterior distribution accounting for action and observation: $p(\boldsymbol{x}_t \mid \boldsymbol{u}_{1:t}, \boldsymbol{z}_{1:t})$
- Goal: Update the posterior distribution over time taking into account controls and measurements such that the extracted estimate converges against the real state

The posterior distribution is the *belief*, $bel(\boldsymbol{x}_t)$ for robot's state:
a probability distribution over all possible robot's states

MARKOV ASSUMPTION

Markov assumption

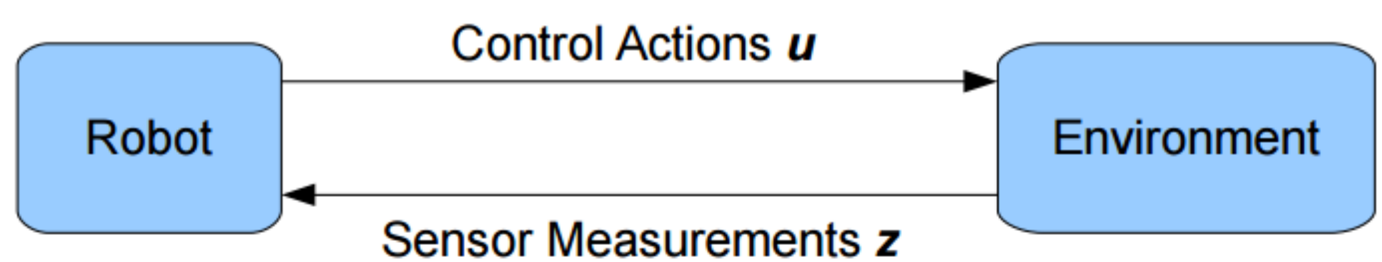


$$p(z_t \mid x_{0:t}, z_{1:t}, u_{1:t}) = p(z_t \mid x_t)$$
$$p(x_t \mid x_{1:t-1}, z_{1:t}, u_{1:t}) = p(x_t \mid x_{t-1}, u_t)$$

State is a sufficient statistics

Static world

Independent noise

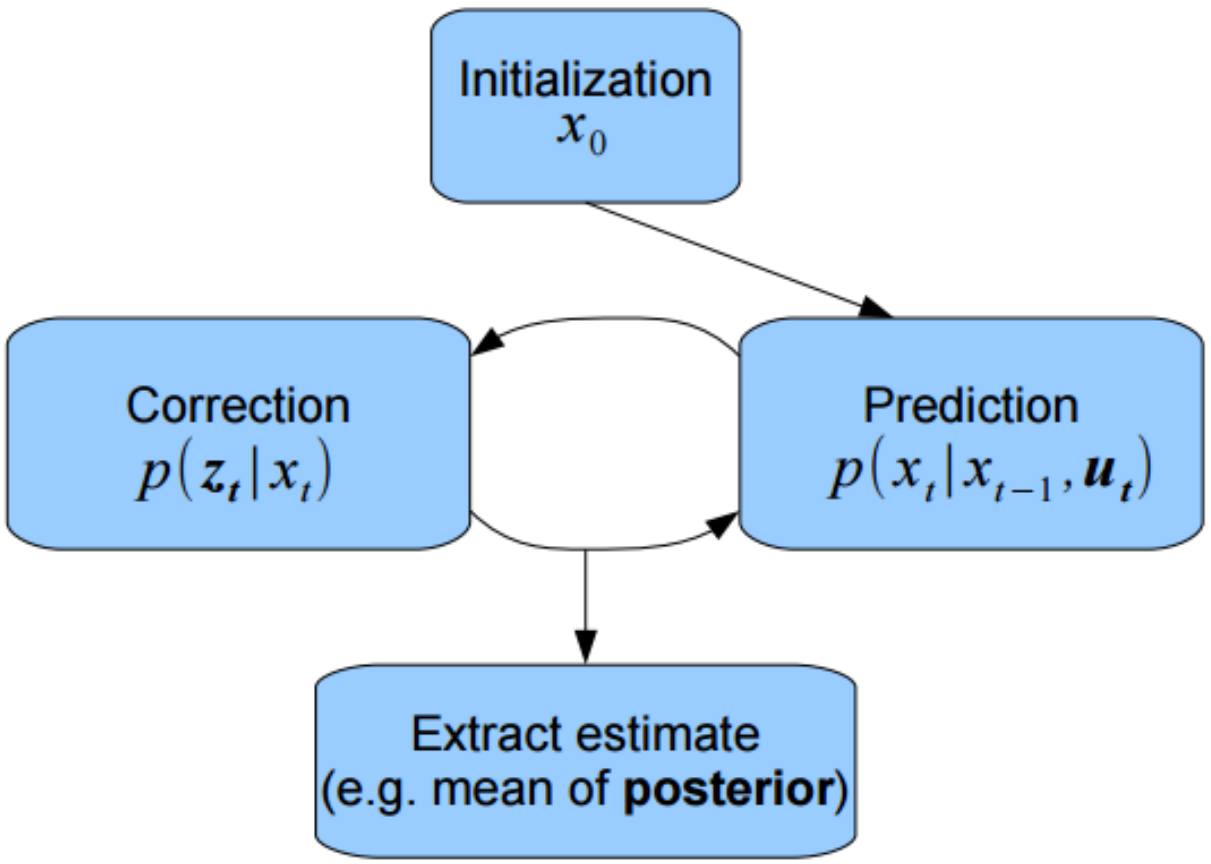


- Initial state: $\boldsymbol{x}_0$
- Posterior distribution: $p(x_t | u_t, z_t)$
- State transition probability: $p(x_t | x_{t-1}, \boldsymbol{u}_t)$

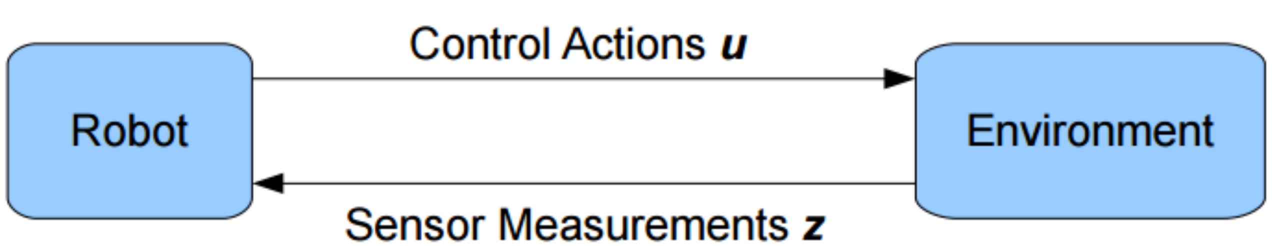
  → *How does state change over time (e.g. as an effect of a control action)?*
- Measurement probability: $p(\boldsymbol{z}_t | x_t)$

  → *Assumed state x_t, how probable is measurement z_t?*

BAYESIAN FILTER



Use <u>Bayes rule</u> for performing Prediction and Correction updates



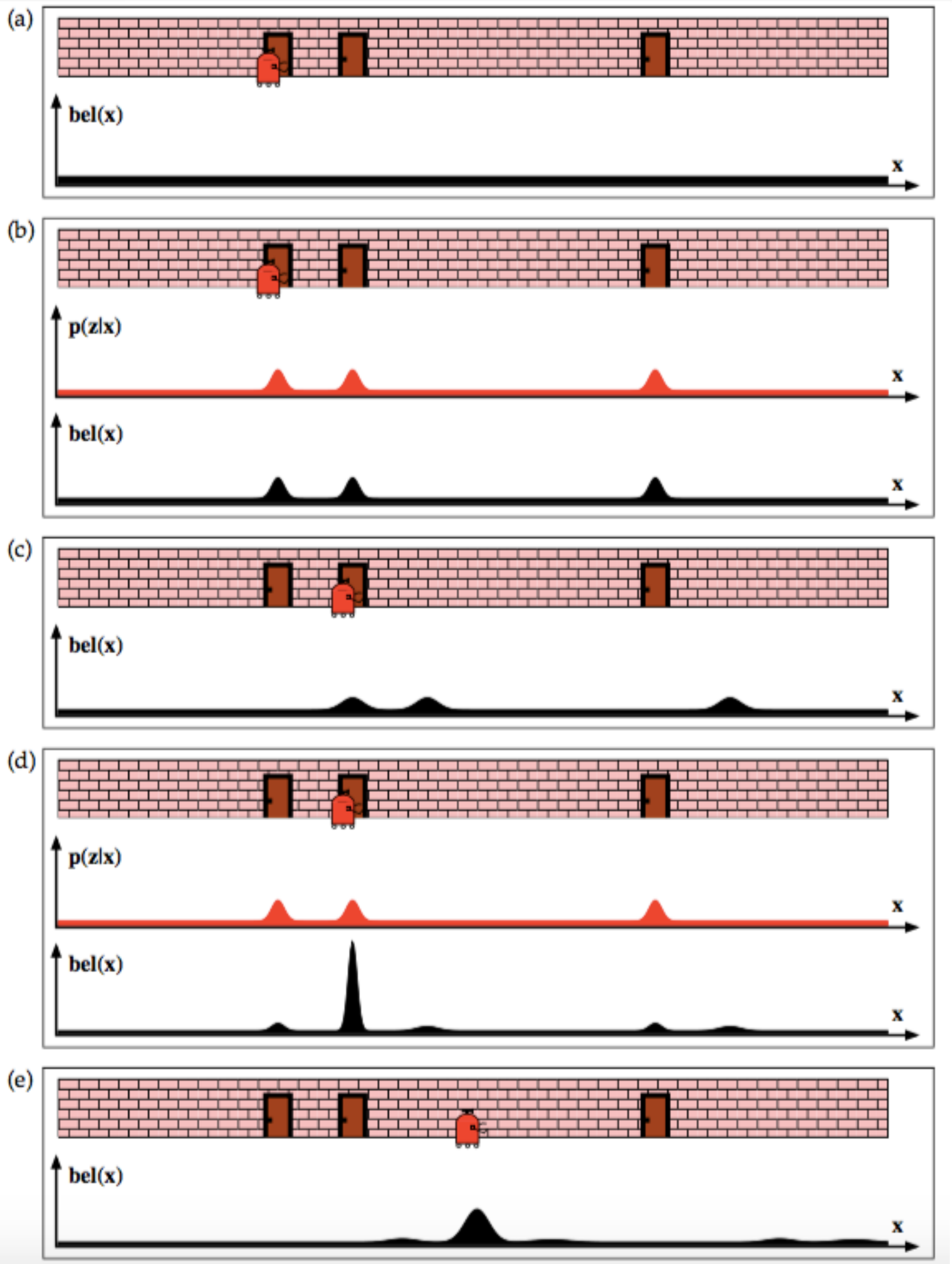
Posterior probability is called the <u>*belief*</u>

- <u>Initial state:</u> x_0
- <u>Posterior distribution:</u> $p(x_t|u_t, z_t)$
- <u>State transition probability:</u> $p(x_t|x_{t-1}, u_t)$

  → *How does state change over time (e.g. as an effect of a control action)?*
- <u>Measurement probability:</u> $p(z_t|x_t)$

  → *Assumed state x_t, how probable is measurement z_t?*

NON-PARAMETRIC VS. GAUSSIAN FILTERS



BAYES FORMULA

$$P(x,y) = P(x \mid y)P(y) = P(y \mid x)P(x)$$

$$\Rightarrow$$

$$P(x \mid y) = \frac{P(y \mid x)\ P(x)}{P(y)} = \frac{\text{likelihood} \cdot \text{prior}}{\text{evidence}}$$

Normalization

$$P(x \mid y) = \frac{P(y \mid x)\ P(x)}{P(y)} = \eta\ P(y \mid x)\, P(x)$$

$$\eta = P(y)^{-1} = \frac{1}{\sum_x P(y \mid x) P(x)}$$

Algorithm

$$\forall x : \text{aux}_{x|y} = P(y \mid x)\ P(x)$$

$$\eta = \frac{1}{\sum_x \text{aux}_{x|y}}$$

$$\forall x : P(x \mid y) = \eta\ \text{aux}_{x|y}$$

TOTAL PROBABILITY AND CONDITIONING

- Total probability:

$$P(x) = \int P(x,z)dz$$

$$P(x) = \int P(x \mid z)P(z)dz$$

$$P(x \mid y) = \int P(x \mid y,z)\, P(z)\, dz$$

- Conditional independence $$P(x,y \mid z) = P(x \mid z)P(y \mid z)$$

equivalent to

$$P(x \mid z) = P(x \mid z,y)$$

and

$$P(y \mid z) = P(y \mid z,x)$$

- **Given:**
  - Stream of observations *z* and action data *u:*

$$d_t = \{u_1, z_1 \ldots, u_t, z_t\}$$

  - Sensor model *P(z|x).*
  - Action model *P(x|u,x').*
  - Prior probability of the system state *P(x).*
- **Wanted:**
  - Estimate of the state *X* of a dynamical system.
  - The posterior of the state is also called **Belief**:

$$Bel(x_t) = P(x_t \mid u_1, z_1 \ldots, u_t, z_t)$$

Markov assumption:

$$p(z_t \mid x_{0:t}, z_{1:t}, u_{1:t}) = p(z_t \mid x_t)$$
$$p(x_t \mid x_{1:t-1}, z_{1:t}, u_{1:t}) = p(x_t \mid x_{t-1}, u_t)$$

NEXT ...

Follow up on the slides from Cyrill Stachniss (check course website):

- Bayes filters
- Particle filter