15-382 COLLECTIVE INTELLIGENCE – S18

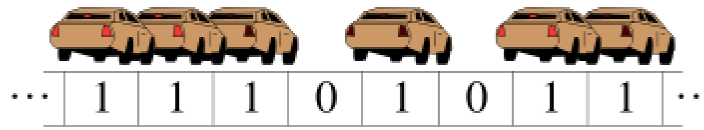
LECTURE 13:
CELLULAR AUTOMATA 3 /
DISCRETE-TIME DYNAMICAL SYSTEMS 5

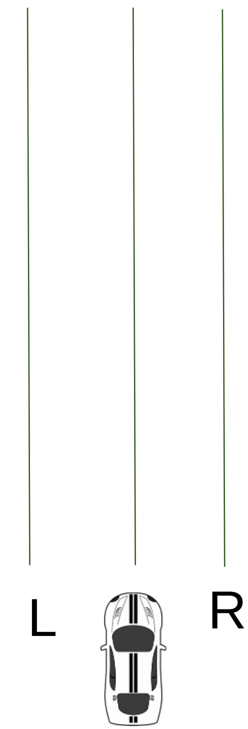
INSTRUCTOR:
GIANNI A. DI CARO

جامعة كارنيجي ميلون في قطر
Carnegie Mellon University Qatar

Rule 184 for car traffic simulation

- Single lane
- Parallel multi-lane






```
Neighborhood state:  111   110   101   100   011   010   001   000
New cell state:       1     0     1     1     1     0     0     0
```

Move right-forward
if space

```
Neighborhood state:  111   110   101   100   011   010   001   000
New cell state:       1     0     1     1     1     0     0     0
```

Move forward if space

CA FOR TRAFFIC SIMULATION: PARTICLE HOPPING MODEL

- According to the properties seen for traffic flow, the behavior for different densities is as follows:
  - $\rho < 0.5$: the CA stabilizes into clusters of cells in state 1, spaced two units apart, with the clusters separated by blocks of cells in state 0. Patterns of this type move rightwards.

0	0	**1**	0	**1**	**1**	0	0
0	0	0	**1**	**1**	0	**1**	0
0	0	0	**1**	0	**1**	0	**1**
1	0	0	0	**1**	0	**1**	0
0	**1**	0	0	0	**1**	0	**1**
1	0	**1**	0	0	0	**1**	0

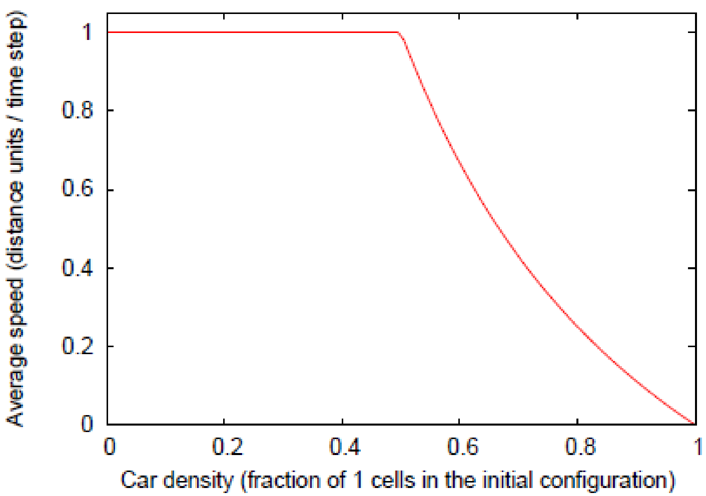
  - $\rho > 0.5$: the CA stabilizes into clusters of cells in state 1, spaced two units apart, with the clusters separated by blocks of cells in state 0. Patterns of this type move leftwards

0	**1**	**1**	0	**1**	**1**	0	**1**
1	**1**	0	**1**	**1**	0	**1**	0
1	0	**1**	**1**	0	**1**	0	**1**
0	**1**	**1**	0	**1**	0	**1**	**1**
1	**1**	0	**1**	0	**1**	**1**	0
1	0	**1**	0	**1**	**1**	0	**1**

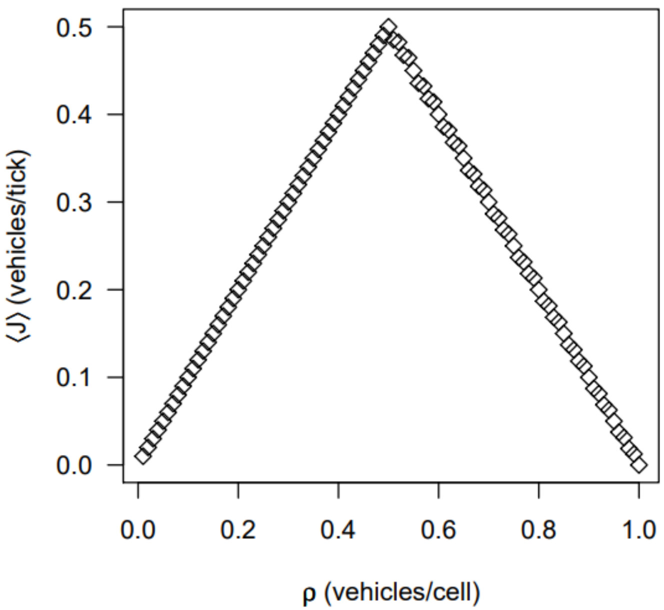
  - $\rho = 0.5$ the initial pattern slowly stabilizes to a pattern that can equivalently be viewed as moving either leftwards or rightwards at each step
- According to this behavior, each cells 'sees' all the 0 and 1 states in the automaton: This can be used to derive the majority status in the entire CA. That is, the CA can be used to detect parity status in input patterns

Rule 184: Phase transition

- Rule 184 as it is can already predict some of the familiar emergent features of real traffic:
  - *clusters of freely moving cars separated by stretches of open road when traffic is light*
  - *waves of stop-and-go traffic when it is heavy*
- Average particle speed:
  - if density is $\rho < 0.5$ the average speed is one unit of distance per unit of time, and after stabilization no car ever slows
  - at $\rho = 0.5$ the system exhibits a second-order kinetic phase transition, with the average speed exponentially decreasing as $\frac{1-\rho}{\rho}$ for $\rho > 0.5$

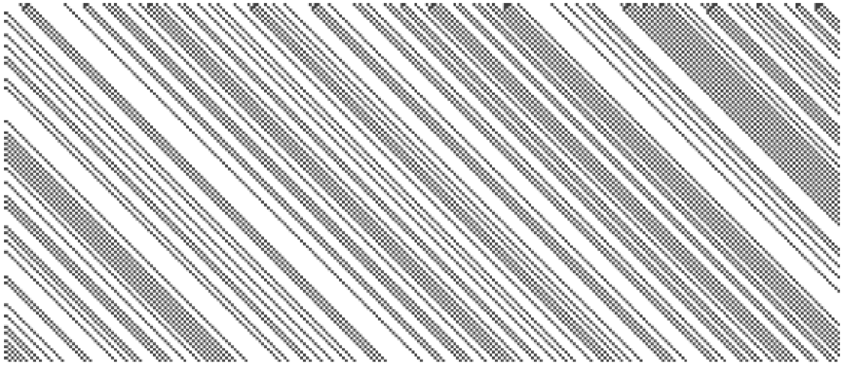


Average flux

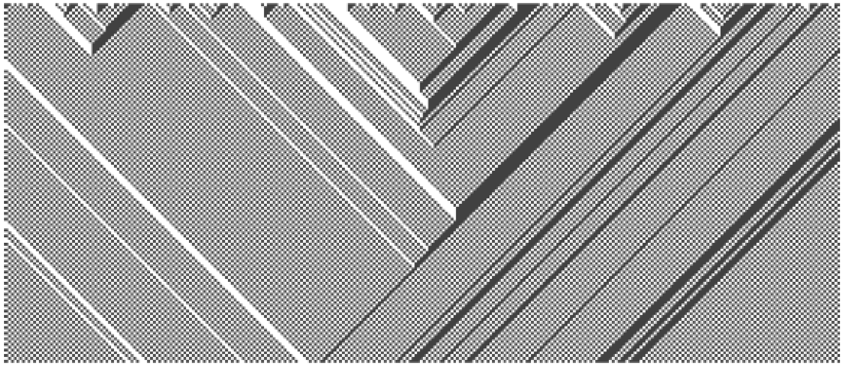


DENSITY-DEPENDING BEHAVIOR

Cars advance one cell per time tick, no jams, the slope is given by the velocity

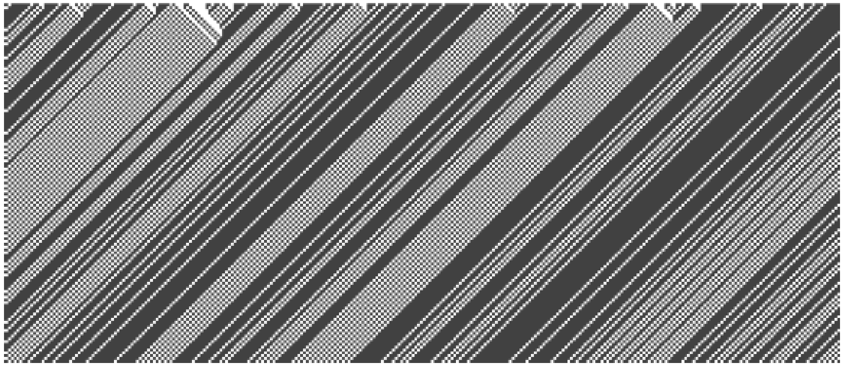


$\rho = 0.25$



$\rho = 0.5$

Cars can only advance when there is space, jams propagates to the left (backwards)



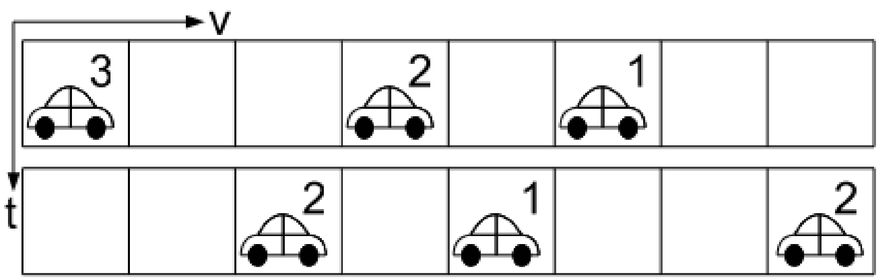
$\rho = 0.75$

NAGEL-SCHRECKENBERG MODEL

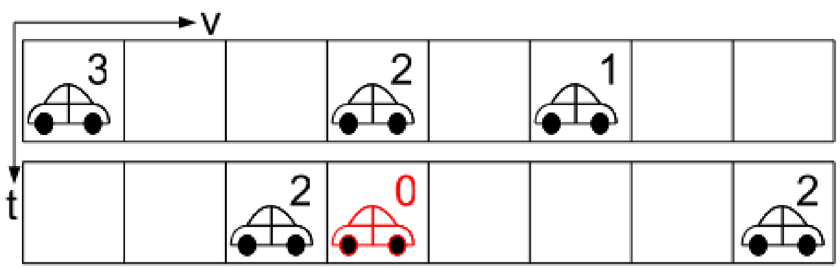
- One-lane, *follower model*, include human (mis)behavaior

1. *Acceleration:* $v_n \rightarrow \min(v_n + 1, v_{max})$
2. *Deceleration:* $v_n \rightarrow \min(v_n, d_n - 1)$
3. *Randomization:* $v_n \rightarrow \max(v_n - 1, 0)$ with probability p
4. *Movement:* $x_n \rightarrow x_n + v_n$

Probabilistic CA!



No randomization



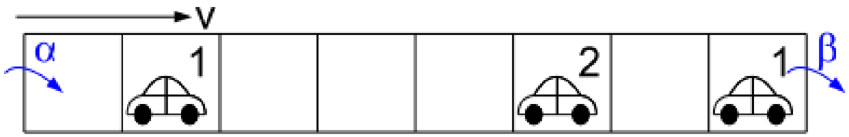
Randomization: basis for *jams*!

- Irreducible model: all four aspects have to be included
- What is the *neighborhood* set? And the evolution function?

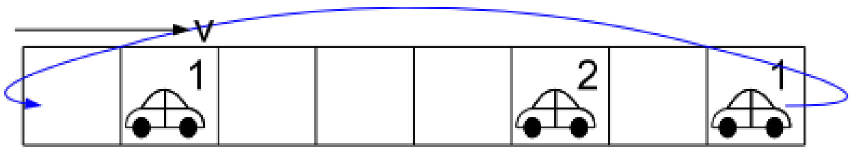
Nagel, K., Schreckenberg, M., *A cellular automaton model for freeway traffic*. Journal de Physique I. 2 (12): 2221, 1992

BOUNDARY CONDITIONS AND PARAMETER SETTING

Open boundaries:
density changes



Periodic boundaries:
density doesn’t change

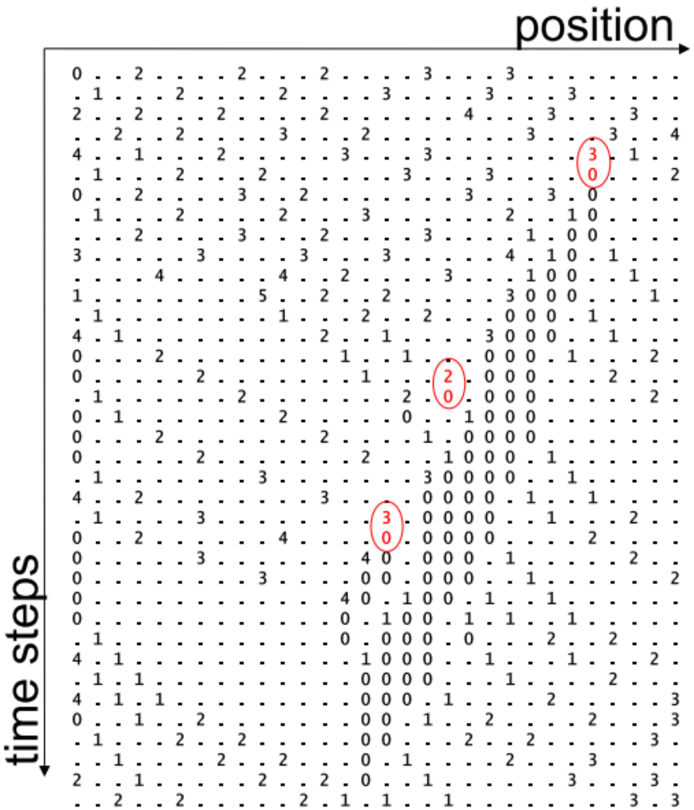


α = Probability for a car entering
β = Probability of exiting (if speed is non-zero at the exit point)

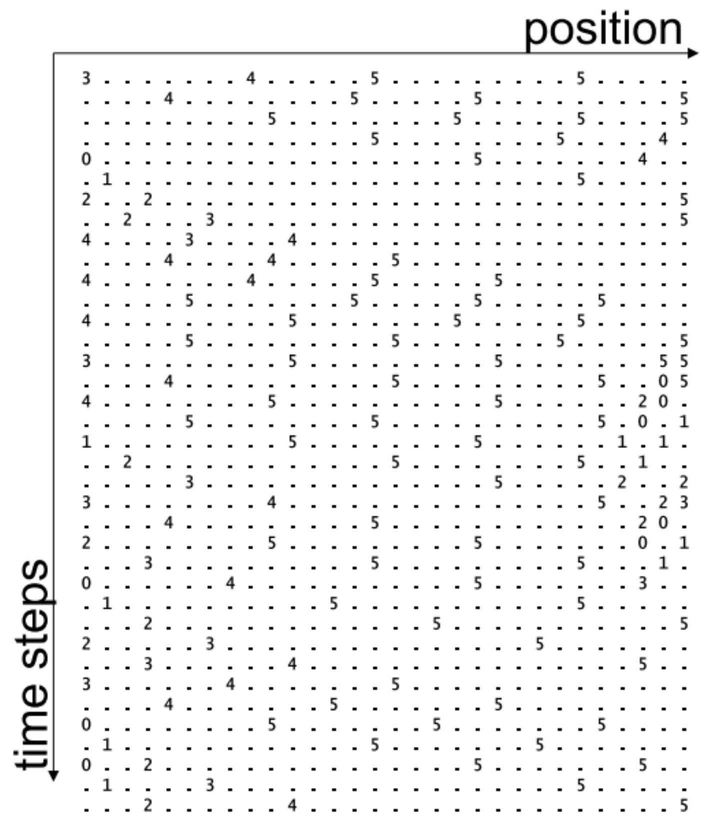
- ~7.5m space for one car → “Width” of a cell
- Reaction time of a driver: ~1 sec → Time step
- Velocity of one cell / per second, $v = 1$ → 27 Km/h
- $v_{max} = 5$ → 135 Km/h, reasonable!

Impact of randomization

$\alpha = 0.3, \beta = 0.8, p = 0.5$, $\mathrm{L} = 30$ cells



$\alpha = 0.3, \beta = 0.8, p = 0$, $\mathrm{L} = 30$ cells



- A dot stands for a free cell
- Numbers are the velocity of a car in the cell as from the last time step
- With randomization, jams are formed, sudden deceleration (e.g., from 3 to 0)
- Without randomization jams only occurs at the exit (because of β, a car may not be entitled to exit the road line)

Velocity-Dependent Randomization (VDR) model

- *Slow-to-start rule*: If a car stops, it takes longer to restart → randomization parameter is higher
- Typical behavior (e.g., at traffic lights), that has dramatic negative impact on flows!

0. *Determination of the randomization parameter:* $p_n = p(v_n)$

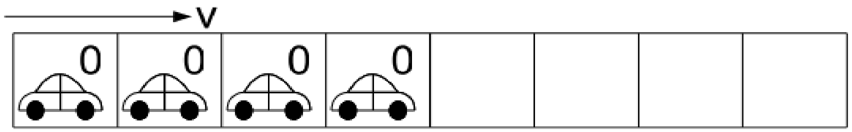
$$p_n = \begin{cases} p_0, & \text{if } v_n = 0 \\ p, & \text{if } v_n > 0 \end{cases}, \text{ with } p_0 > p$$

1. *Acceleration:* $v_n \to \min(v_n + 1, v_{max})$
2. *Deceleration:* $v_n \to \min(v_n, d_n - 1)$
3. *Randomization:* $v_n \to \max(v_n - 1, 0)$ with probability p_n
4. *Movement:* $x_n \to x_n + v_n$

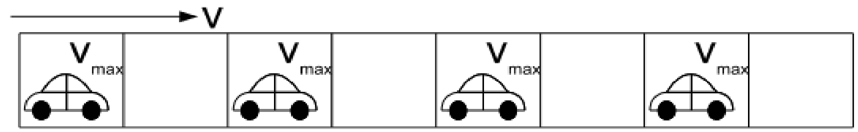
- *Cruise control* (at v_{max} no human ctrl): $p(v_{max}) = 0,\ p(v) = p$ for $v < v_{max}$

A. Clarridge and K. Salomaa, *Analysis of a cellular automaton model for car traffic with a slow-to-stop rule*, Theoretical Computer Science, vol. 411, no. 38-39, pp. 3507–3515, 2010.

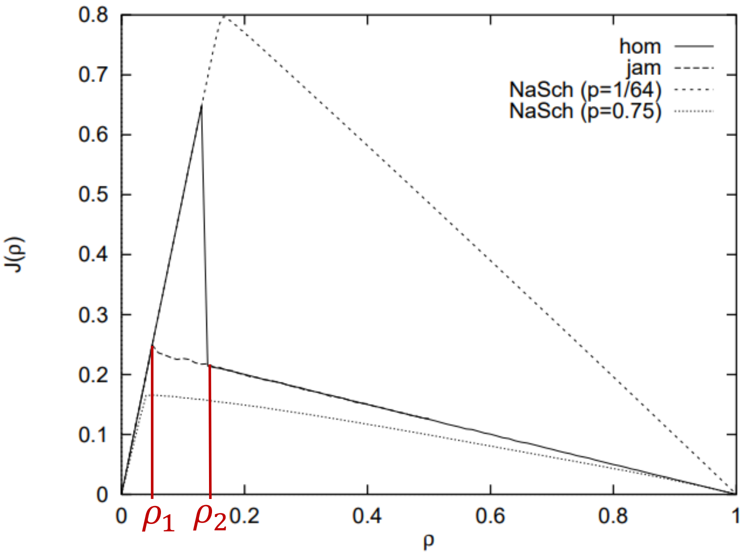
Phase transition and Metastability



Starting jam



Optimal, homogeneous start

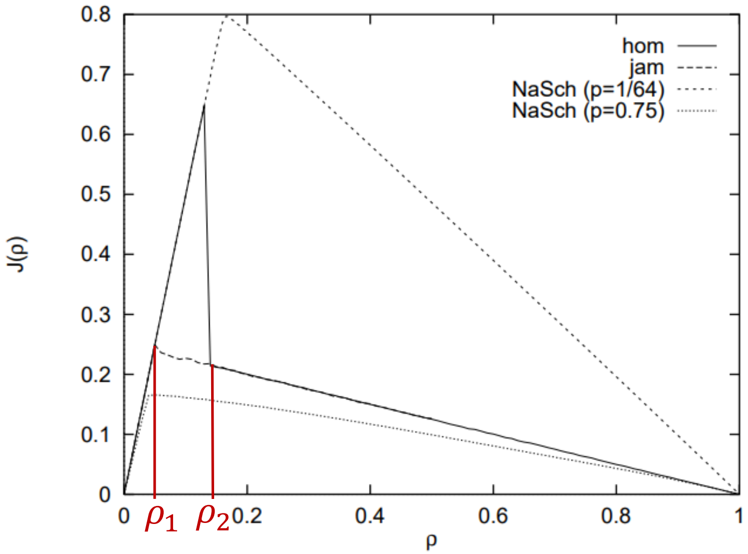


$$v_{max} = 5, p_0 = 0.75, p = 1/64, L = 10000$$

- Free flow phase: for low densities, flow increases linearly with density
- Phase transition: At a critical density, flows experience a sudden jammed state, then keep decreasing linearly, jam doesn't disperse
- For the jammed start case, the initial jam can't really disperse

- Metastability: For the same values of ρ in $[\rho_1, \rho_2]$, two equilibrium states are possible depending on initial conditions. For the homogeneous condition, the critical density defines a metastable equilibrium collapsing into a jammed state
- Basic NaSch model with randomization parameter p low does not lead to a stable jam and has regular linear behavior. High p values result in very low flows

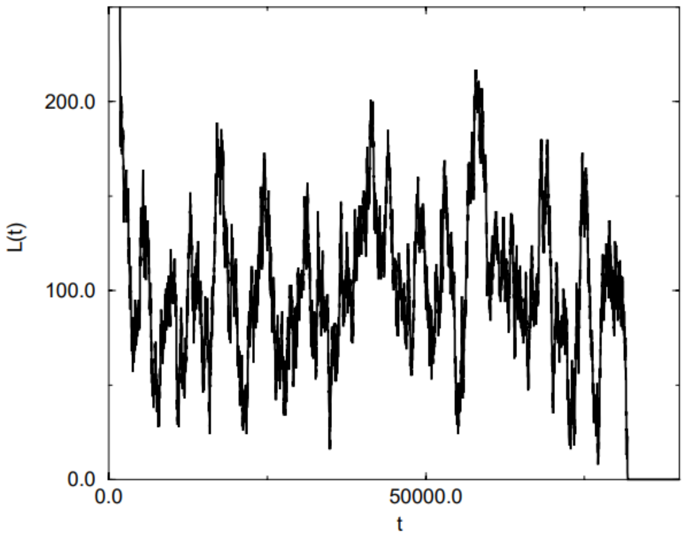
ANALYSIS OF THE SYSTEM



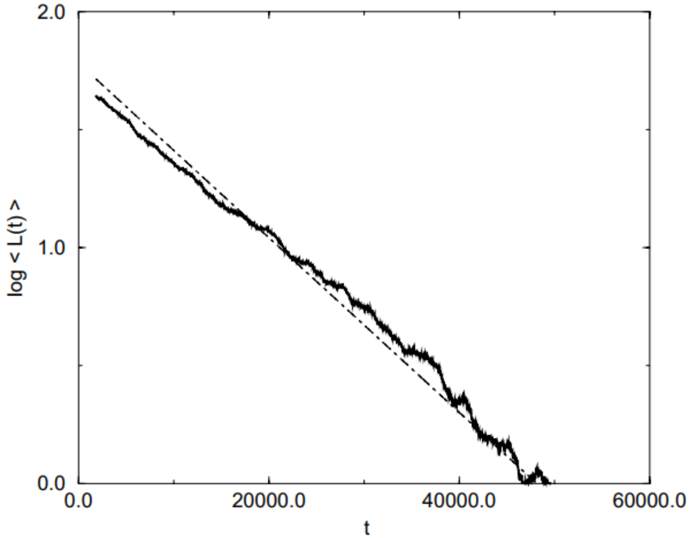
- For low densities, there are no slow cars, since interactions are rare, flows go as: $J(\rho) \approx \rho(v_{max} - p)$
- For large densities, flows go as: $J(\rho) \approx (1 - p_0)(1 - \rho)$ that corresponds to the NaSch model with randomization p_0
- For $\rho \approx 1$ only cars with $v = 0$ or $v = 1$ exist
- The flow goes asymptotically to zero, with a rate being determined by p_0

R. Barlovic, L. Santen, A. Schadschneider, M. Schreckenberg, *Metastable states in cellular automata for traffic flow*, The European Physical Journal B - Condensed Matter and Complex Systems, Volume 5, Issue 3, pp 793–800, October 1998

LIFETIME OF THE METASTABLE PHASE



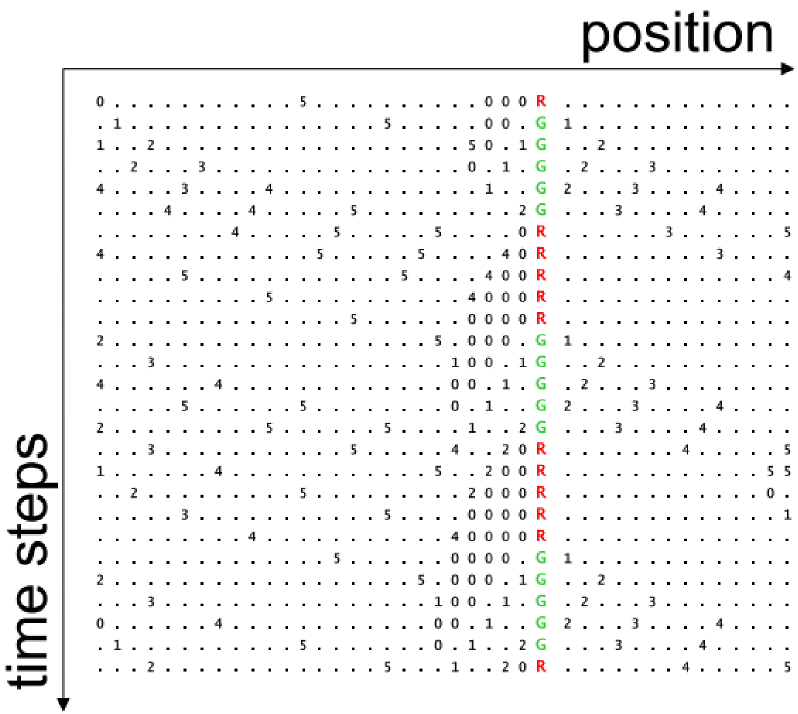
Time-dependent length $L_{jam}(t)$ of initial jam for one run, $\rho = 0.095$

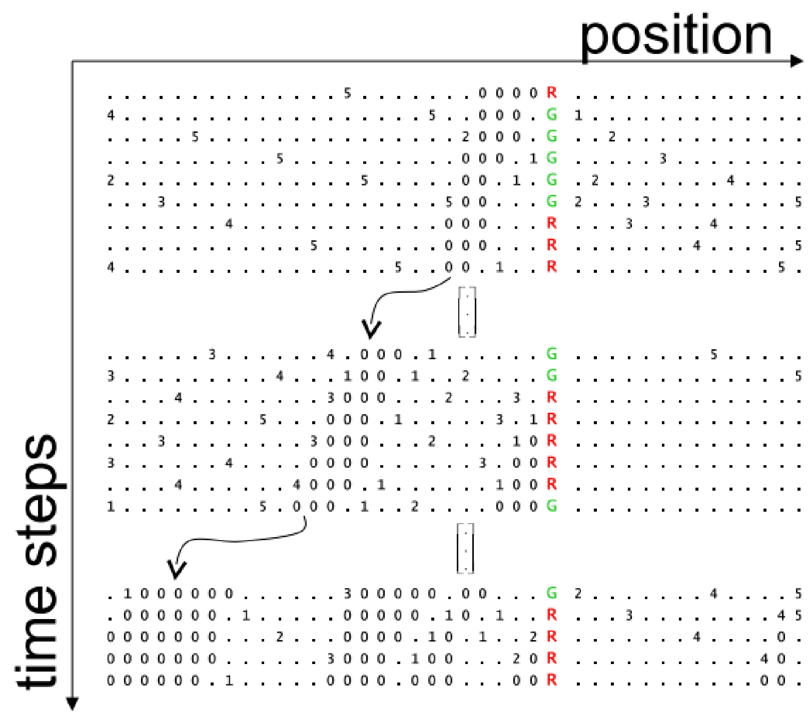


$< L_{jam}(t) >$ over 10,000 samples (in log scale)

- For the jammed start, close to ρ_1, the large jam present in the initial configuration dissolves and the average length decays exponentially in time (linear in log-scale) through fluctuations without any obvious systematic time-dependence
- Once a homogeneous state without a jammed car is reached, no new jams are formed. Therefore the homogeneous state is stable near ρ_1
- For homogeneous start, for $\rho \gtrsim \rho_2$, metastable homogeneous states are created with short lifetime

EFFECT OF TRAFFIC LIGHTS

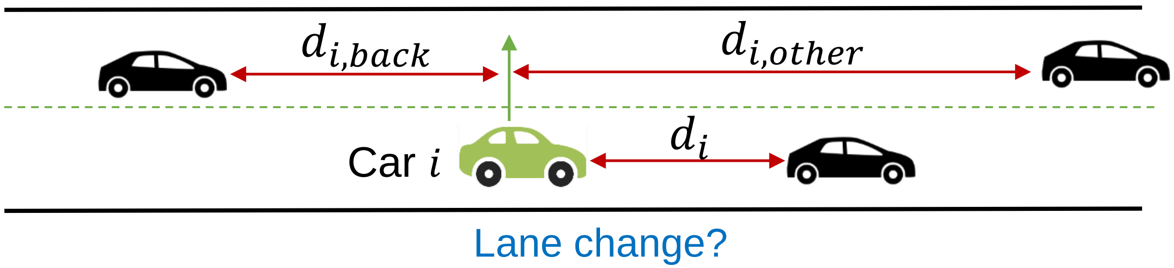




- In the basic NaSch model, jams form in front of the red traffic lights, but vanish again in the green phases.
- In VDR model the jams persist and start to move backwards against the driving direction of the cars, even in the green phases. This is due to the slow-to-start rule.

Rickert-Nagel-Schreckenberg (RNS) model with lane changes

- The single lane model can only result, in the best case, in *platooning* behind the slow cars
- Space permitting, a two-lane model allows to change lane, space permitting, and then possibly overtake the slow car
- It can be designed as two parallel, communicating 1D models, or as a 2D model (with boundary conditions only to left and right sides)



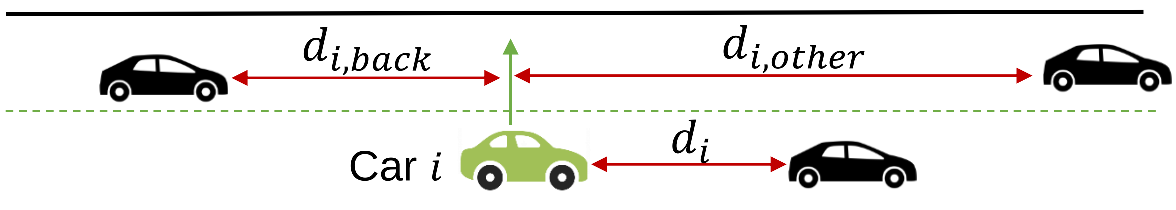
<u>Change lane if:</u>

- **Incentive:** $d_i < \min(v_i + acc, v_{max})$
- \+ **Improvement:** $d_{i,other} > d_i$
- \+ **Safety:** $d_{i,back} > v_{max}$

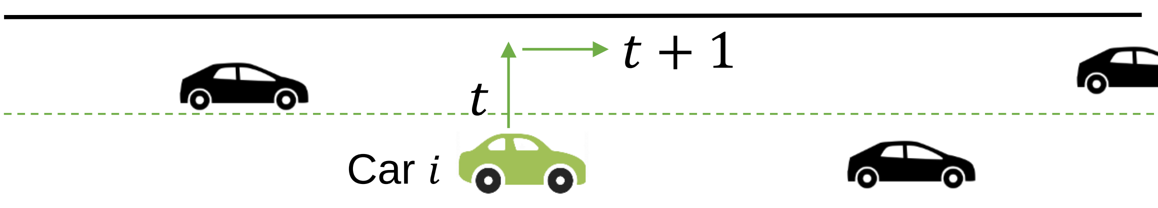
M. Rickert, K. Nagel, M. Schreckenberg, A. Latour. *Two lane traffic simulations using cellular automata.* Physica A: Statistical and theoretical physics, vol. 231, issue 4, 1, pp. 534-550, 1996.

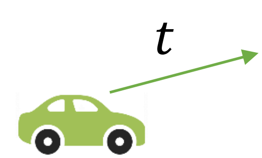
RICKERT-NAGEL-SCHRECKENBERG (RNS) MODEL WITH LANE CHANGES

- Lane change for a car in cell i happens <u>in two time steps</u> given that all four conditions are met:
  - The car is moved to the other line: a 1 appears on cell i of the other lane
  - Next step, car i moves as usual according to NS model
- Apart from lane changing, all cars move according to the NS model
- *No diagonal movement*



Lane change?





No!