



15-382 COLLECTIVE INTELLIGENCE – S18

LECTURE 14:

CELLULAR AUTOMATA 4 /

DISCRETE-TIME DYNAMICAL SYSTEMS 5

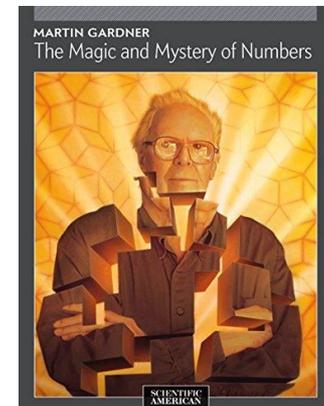
INSTRUCTOR:

GIANNI A. DI CARO

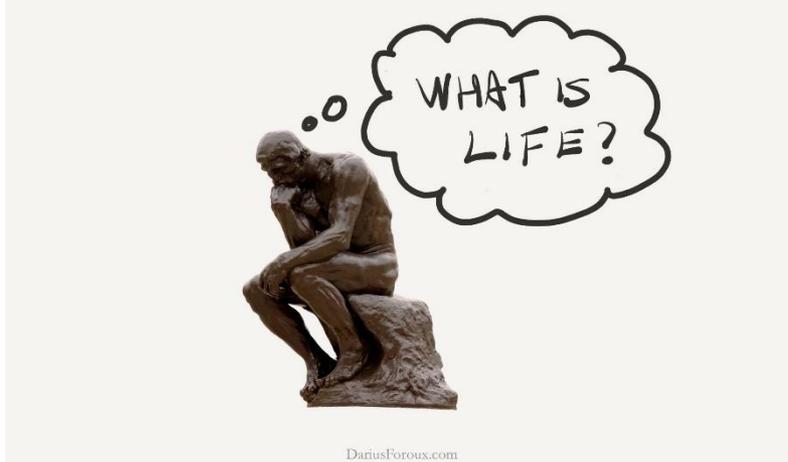
جامعة كارنيجي ميلون في قطر
Carnegie Mellon University Qatar

CONWAY'S GAME OF LIFE

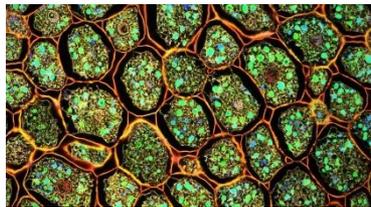
- The Game of Life was invented in 1970 by the British mathematician **John H. Conway**.
- Conway developed an interest in the problem that made **John von Neumann** to define CA: to find a hypothetical machine that has the ability to create copies of itself and live
- Conway's took this original idea on and developed a 2D CA that lives on regular lattice grid regular grid
- **Martin Gardner** popularized the Game of Life by writing two articles for his column "Mathematical Games" in the journal Scientific American in 1970 and 1971.



LIFE?



- What properties do we expect?
- What dynamics?
- → Which local rules?



Organic matter



Inorganic matter

Organization
(structure and function)

Metabolism
(use energy to support
structures and functions)

Homeostasis
(internal regulation)

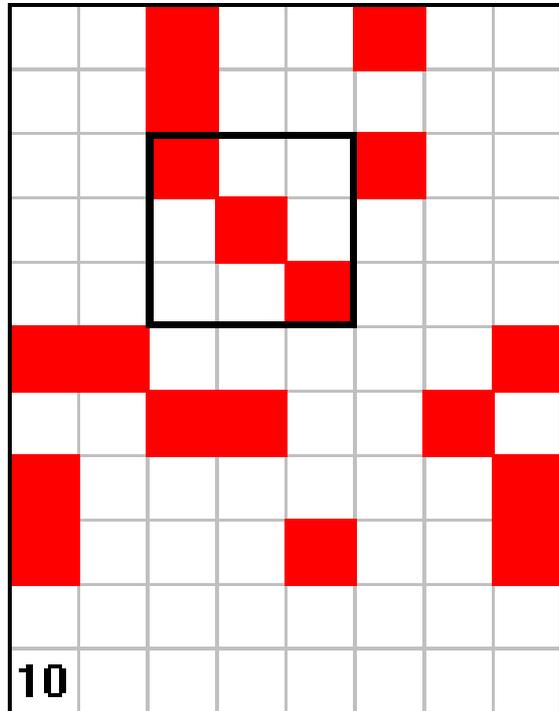
Growth
(change structures)

Reproduction
(to have a population)

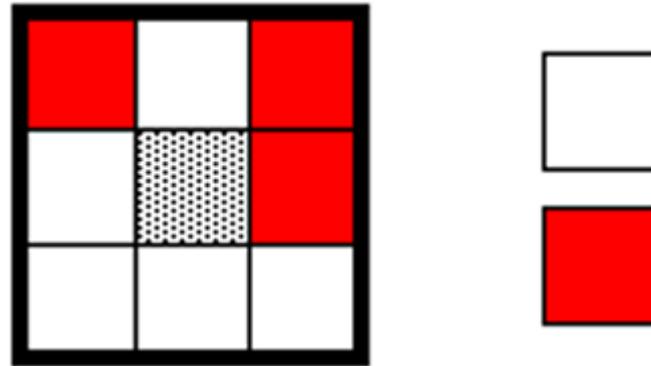
Response
(adapt, react)

Evolution
(phylogenetic adaptation)

CONWAY'S GAME OF LIFE



- 2D regular lattice of identical cells
- **Neighborhood (Moore):** 8 surrounding cells
- Cells are in **two states**: dead or alive

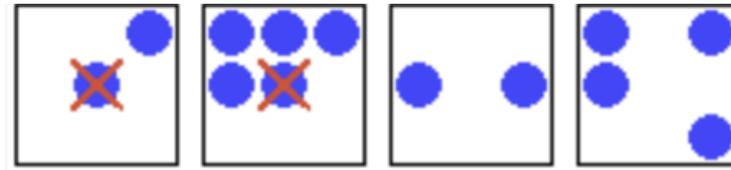


Transition rules:

- *Die* because of overcrowding
- *Die* because of loneliness
- *Keep alive* when in an healthy environment
- *Reproduce* when conditions are favorable

EVOLUTION RULES

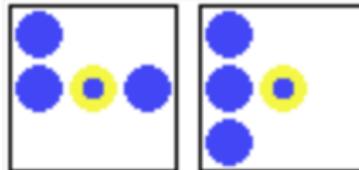
- Any live cell with fewer than two live neighbors dies, as if caused by underpopulation (a few resources) or loneliness
- Any live cell with more than three live neighbors dies, as if by overcrowding



- Any live cell with two or three live neighbors lives on to the next generation

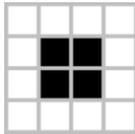


- Any empty / dead cell with exactly three live neighbors becomes alive on to the next generation, as if because of good conditions for reproduction

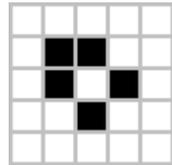


STILL LIFE

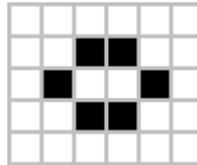
- **Some patterns (local configurations) are *stable*:** do not change if the surrounding environment does not change significantly and can be used to build critical solid parts of more complex patterns
- These patterns stay in one state which enables them to store information or act as solid bumpers to stop other patterns or keep other unstable patterns stable.
- Examples of *still* life include:



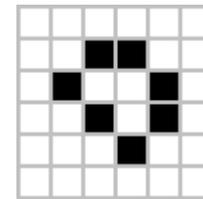
Block



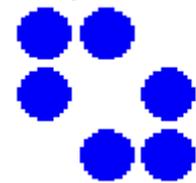
Boat



Loaf



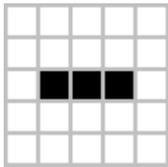
Beehive



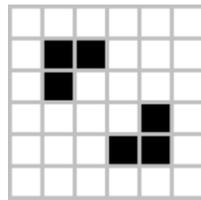
Ship

PERIODIC LIFE FORMS / OSCILLATORS

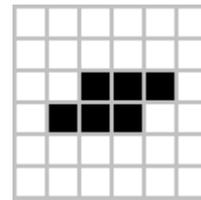
- Some patterns change over a specific number of time steps. If left undisturbed, they repeat their pattern infinitely
- The basic oscillators *have periods of two or three*, but complex oscillators have been discovered with periods of twenty or more
- These oscillators are very useful for setting off other reactions of bumping stable patterns to set off a chain reaction of instability.
- The most common period-2 oscillators include:



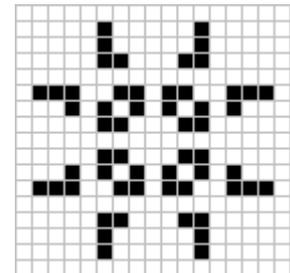
Blinker



Beacon



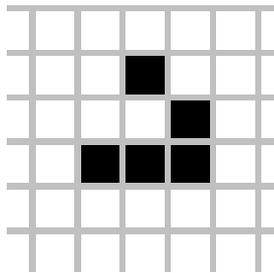
Toad



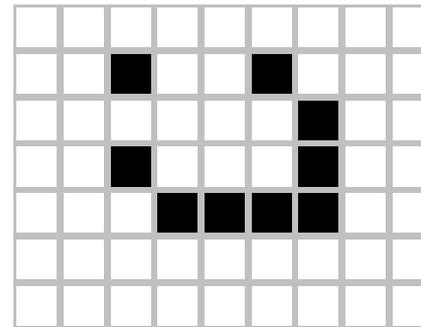
Pulsar

GLIDERS AND SPACESHIPS

- A **spaceship** is a pattern that moves, returning to the same configuration but shifted, after a finite number of generations
- A **glider** is an example of a simple spaceship made of a 5-cell pattern that repeats itself every *four generations*, and moves *diagonally one cell* by time step. It moves at one-quarter the speed of light.
- Other examples of simple spaceships include lightweight, medium weight, and heavyweight spaceships. They each move in a straight line at half the speed of light.



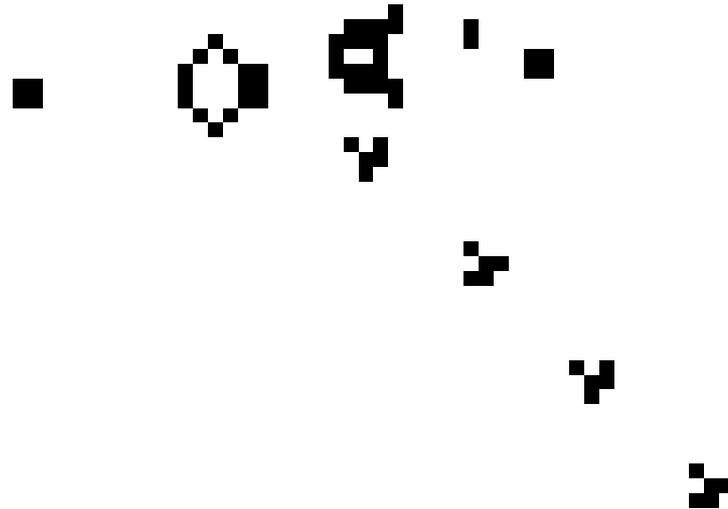
Glider



Lightweight spaceship

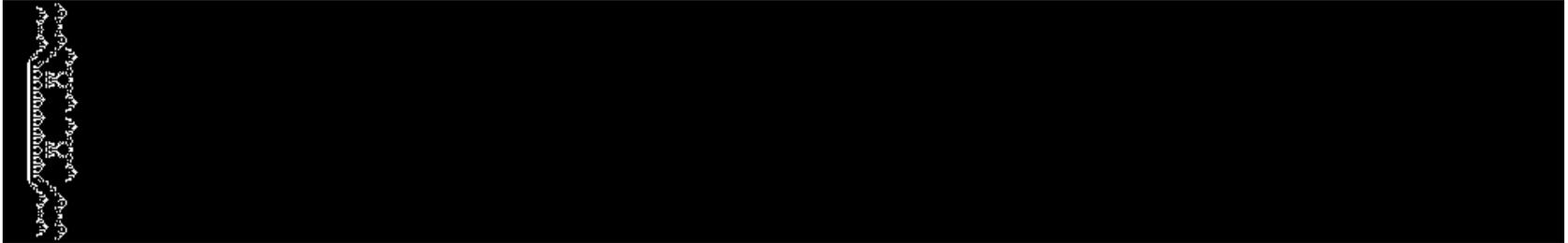
GUNS

- **Guns** are repeating patterns which produce (shoot) a spaceship after a finite number of generations.
- The first discovered gun, called the **Gosper glider gun**, produces a glider every 30 generations. This fascinating pattern was discovered in 1970 by Bill Gosper. Through careful analysis and experimental testing he developed a pattern which emitted a continuous stream of gliders

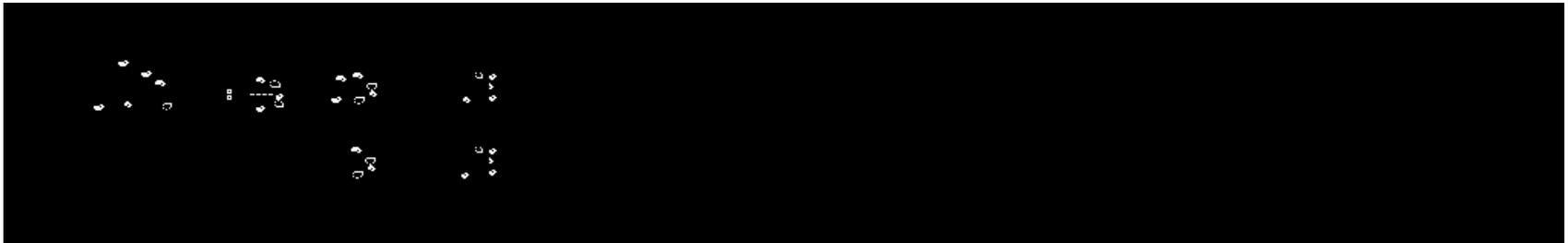


OTHER CREATURES ...

- **Puffer Train or "Puffers"**. Moving patterns whose creation leaves a stable or oscillating debris behind at regular intervals.



- **Rakes**. Moving patterns that emit spaceships at regular intervals as they move.

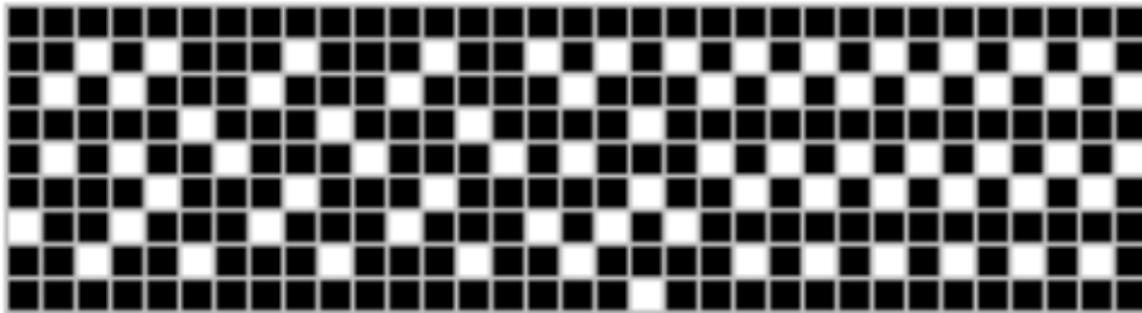


- **Breeder**. Complicated oscillating patterns which leave behind guns at regular intervals. Unlike guns, puffers, and rakes, each with a linear growth rate, breeders have a quadratic growth rate



GARDEN OF EDEN

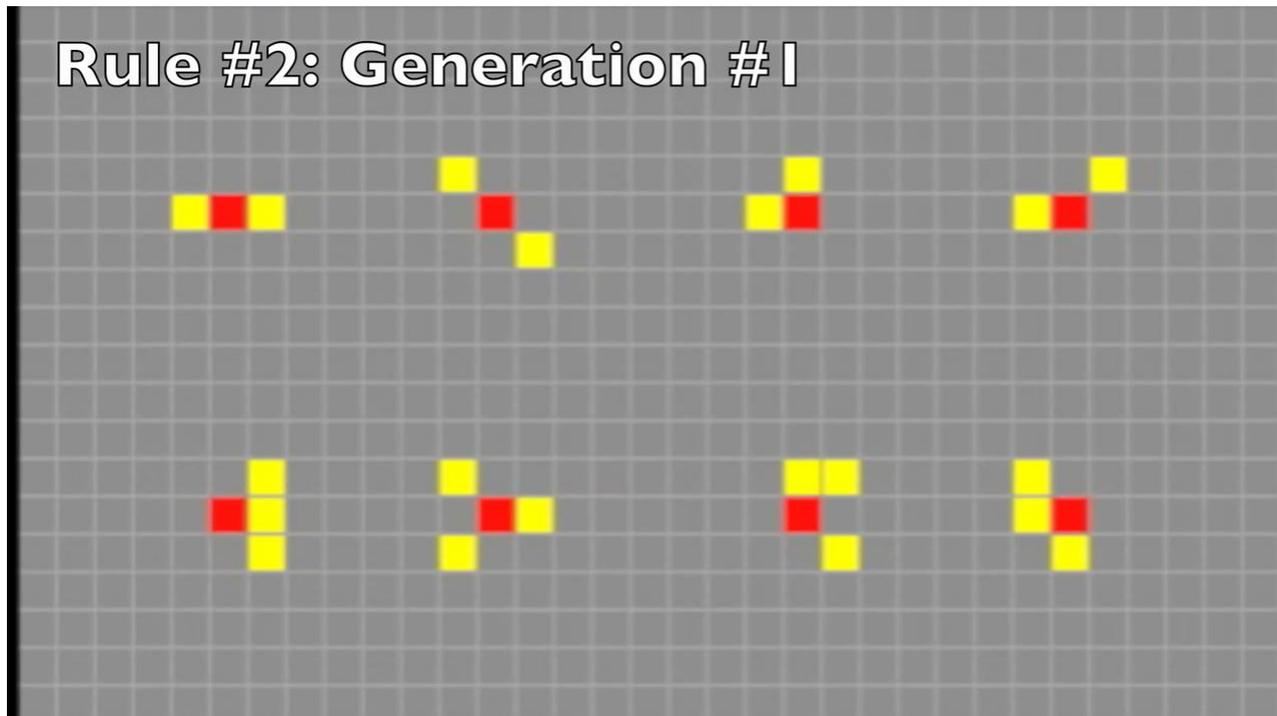
- **Garden of Eden:** A pattern that can only exist *as initial pattern*. In other words, no parent could possibly produce the pattern.



DOES LIFE STOP?

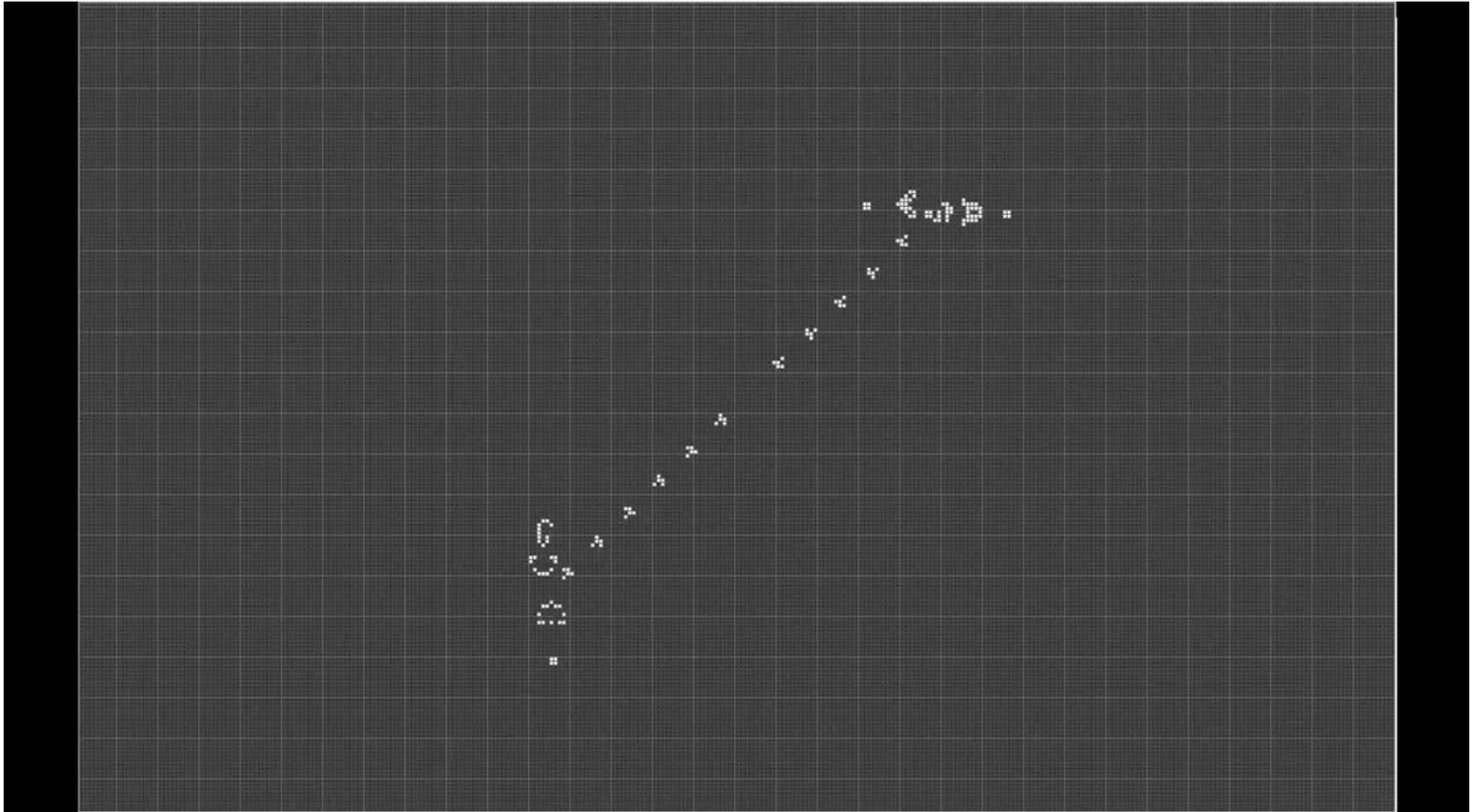
- It is not immediately obvious whether a given initial Life pattern can grow indefinitely, or whether any pattern at all can.
- Conway offered a \$50.00 prize to whoever could settle this question.
- In 1970 an MIT group headed by R.W. Gosper won the prize by finding the glider gun that emits a new glider every 30 generations. Since the gliders are not destroyed, and the gun produces a new glider every 30 generations indefinitely, the pattern grows forever, and thus proves that **there can exist initial Life patterns that grow infinitely.**
- At which max speed life can proceed? → Information propagate?
 - Speed of light, c !
 - The glider takes 4 generations to move one cell diagonally, and so has a speed of $c/4$
 - The light weight spaceship moves one cell orthogonally every other generation, and so has a speed of $c/2$
 - *No spaceships can move faster than glider or light weight spaceship*

RULES IN ACTION



<https://www.youtube.com/watch?v=0XI6s-TGzSs>

CONWAY'S GAME OF LIFE: AMAZING BEHAVIORS



<https://www.youtube.com/watch?v=C2vgICfQawE&t=197s>

GAME OF LIFE: COLLECTION OF LIFE FORMS



<https://www.youtube.com/watch?v=9klgfBsjMuQ&t=56s>

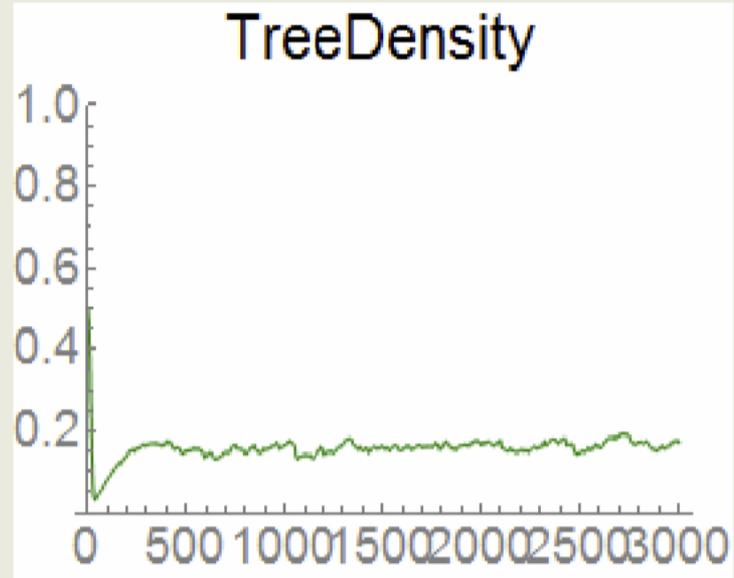
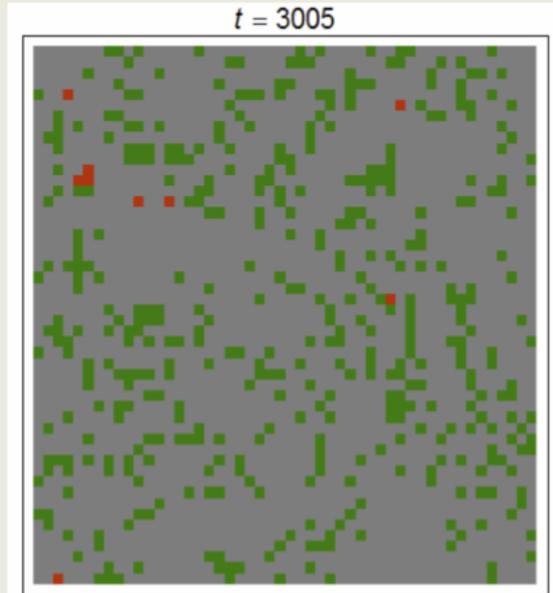
FOREST FIRE MODEL

Forest Fire Model

- Forest Fire Model is a stochastic 3-state cellular automaton defined on a 2-dimensional lattice of size L
- Each site is occupied by a tree, a burning tree, or is empty
- During each time step the system is updated according to the rules:
 - 1) empty site (state 0) \rightarrow tree (state 1): with the growth rate probability p_{growth}
 - 2) tree (state 1) \rightarrow burning tree (state 2): with the lightning rate probability $p_{\text{lightning}}$
 - 3) tree (state 1) \rightarrow burning tree (state 2): with the probability $1 - (1 - p_{\text{catchfire}})^{\text{\#burning neighbors}}$
 - 4) burning tree (state 2) \rightarrow empty site (state 0) with extinction probability p_{ext}

FOREST FIRE MODEL

Forest Fire Model

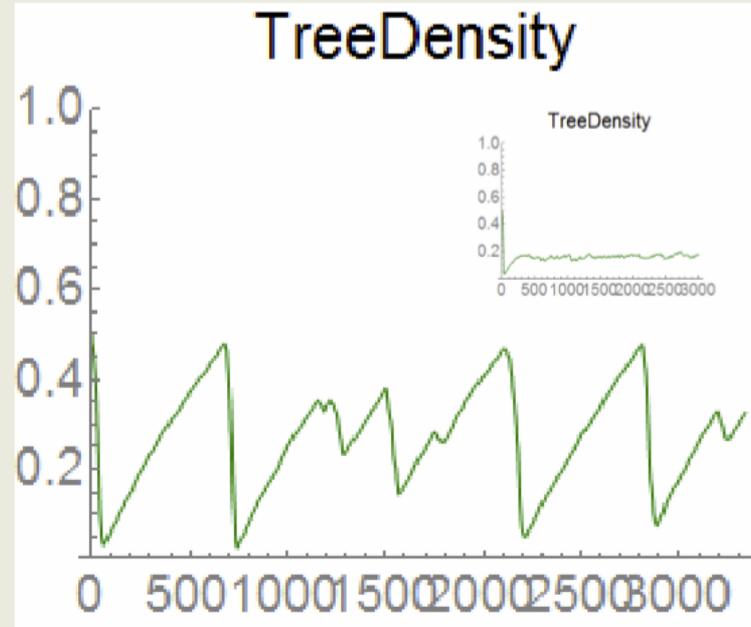
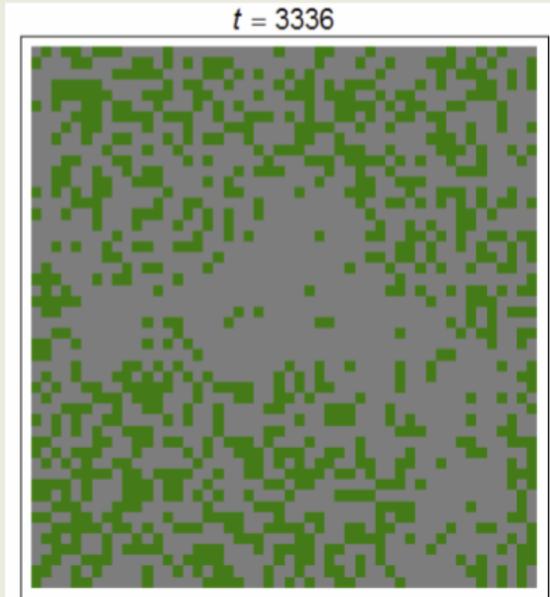


$$p_{lightning} = 10^{-3}$$
$$p_{growth} = 0.001$$
$$p_{catch} = 0.5$$
$$p_{extinct} = 0.2$$

What happens, if the lightning probability is reduced?

FOREST FIRE MODEL

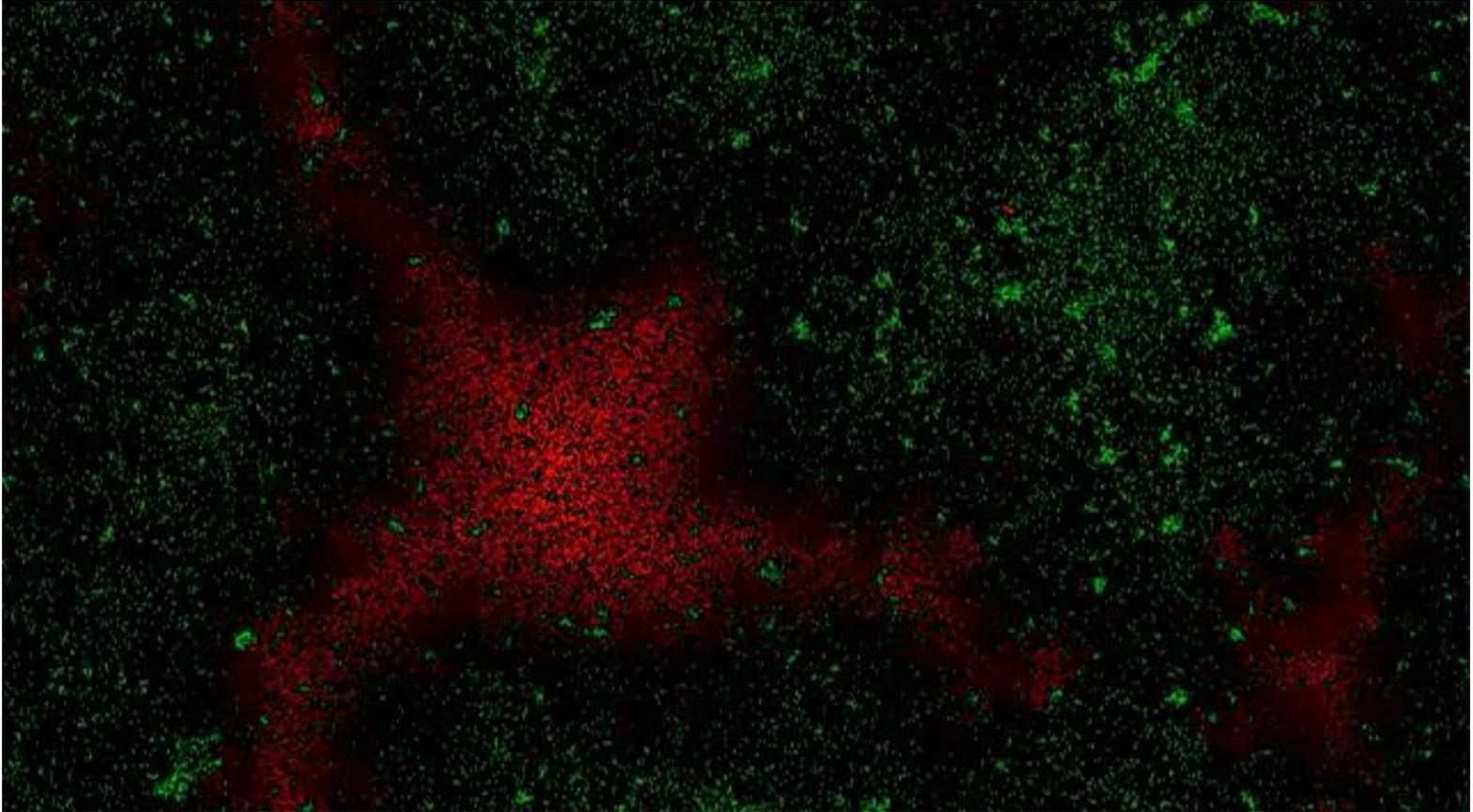
Forest Fire Model



$$p_{\text{lightning}} = 10^{-5}$$
$$p_{\text{growth}} = 0.001$$
$$p_{\text{catch}} = 0.5$$
$$p_{\text{extinct}} = 0.2$$

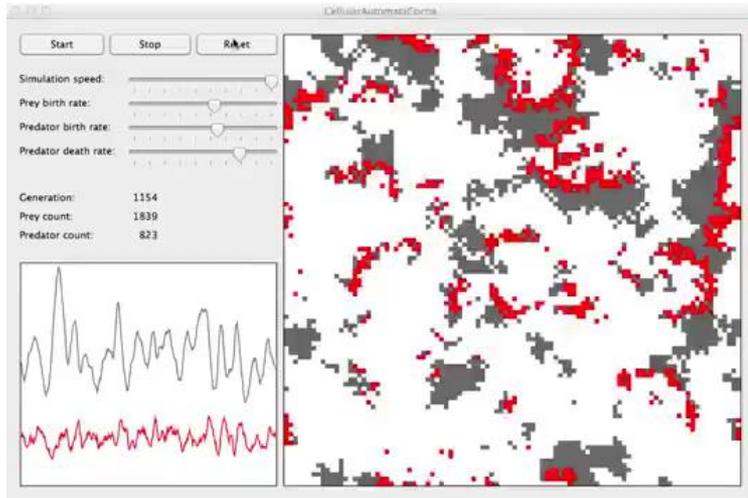
→ Small lightning probabilities lead to higher ecological damage!

A FOREST FIRE MODEL IN ACTION



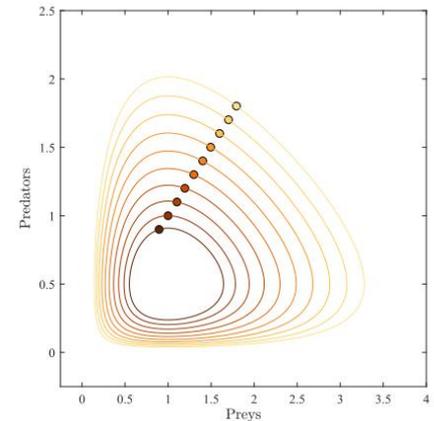
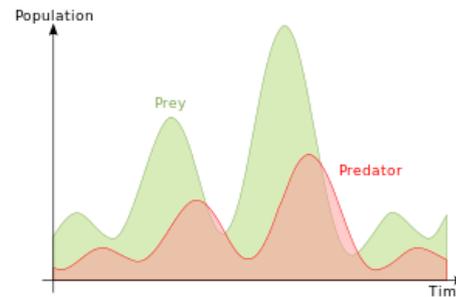
<https://www.youtube.com/watch?v=bUd4d8BDIzI&t=19s>

PREY-PREDATOR MODEL



$$\frac{dx_1}{dt} = g_1 x_1 - i_{21} x_1 x_2$$

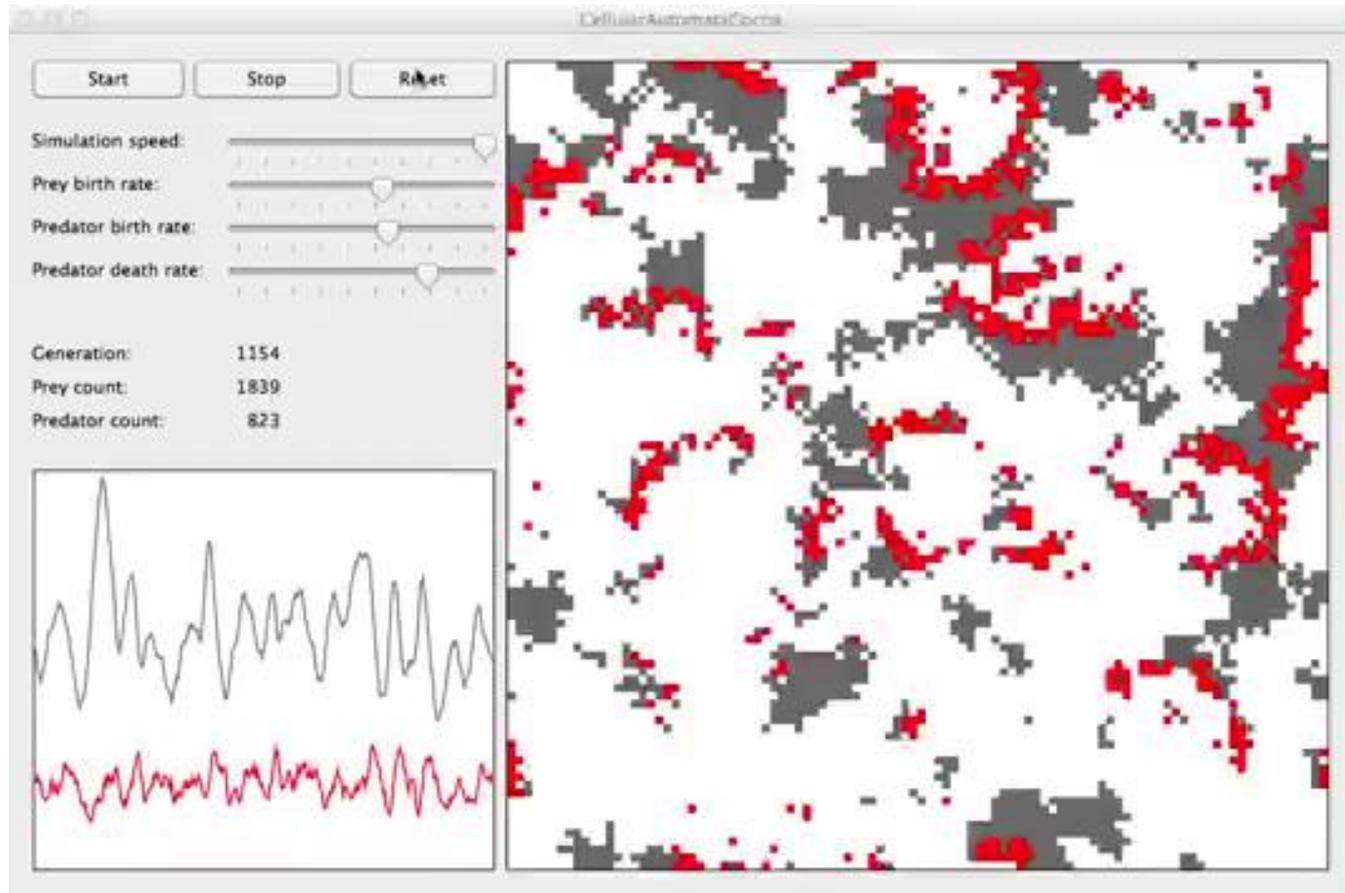
$$\frac{dx_2}{dt} = -g_2 x_2 + i_{12} x_1 x_2$$



Compare it with *the Lotka-Volterra continuous-time differential model*:

- Discrete-time → Integration of infinitesimal variations
- Spatial lattice: the environment where the populations live is introduced, spatial locality is used instead of population-level quantities
- Great flexibility choosing the local (in space, per individual) rules vs. the complexity of the mathematical modeling of coupled interactions

PREY-PREDATOR MODEL

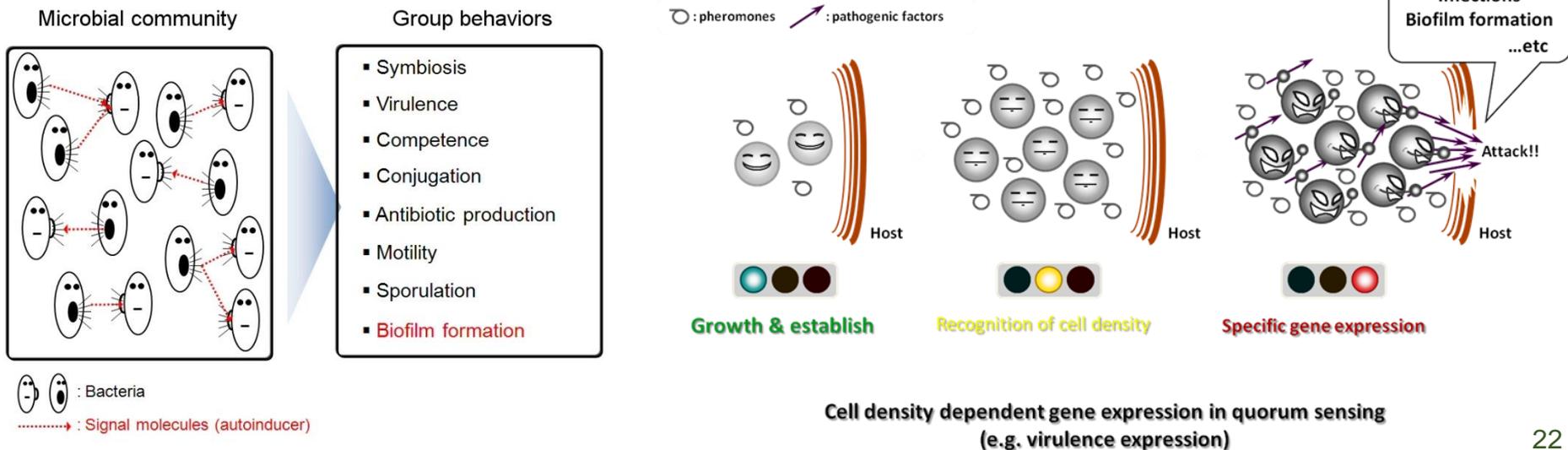


https://www.youtube.com/watch?v=sGKiTL_Es9w&t=51s

G. Cattaneo, A. Dennunzio, F. Farina, *A full Cellular Automaton to simulate predator-prey systems*, Proc. of ACRI, LNCS 4173, 2006

ROCK-PAPER-SCISSORS AUTOMATA: SIMULATION OF BACTERIAL DIFFUSION (BACTERIAL COMPUTING)

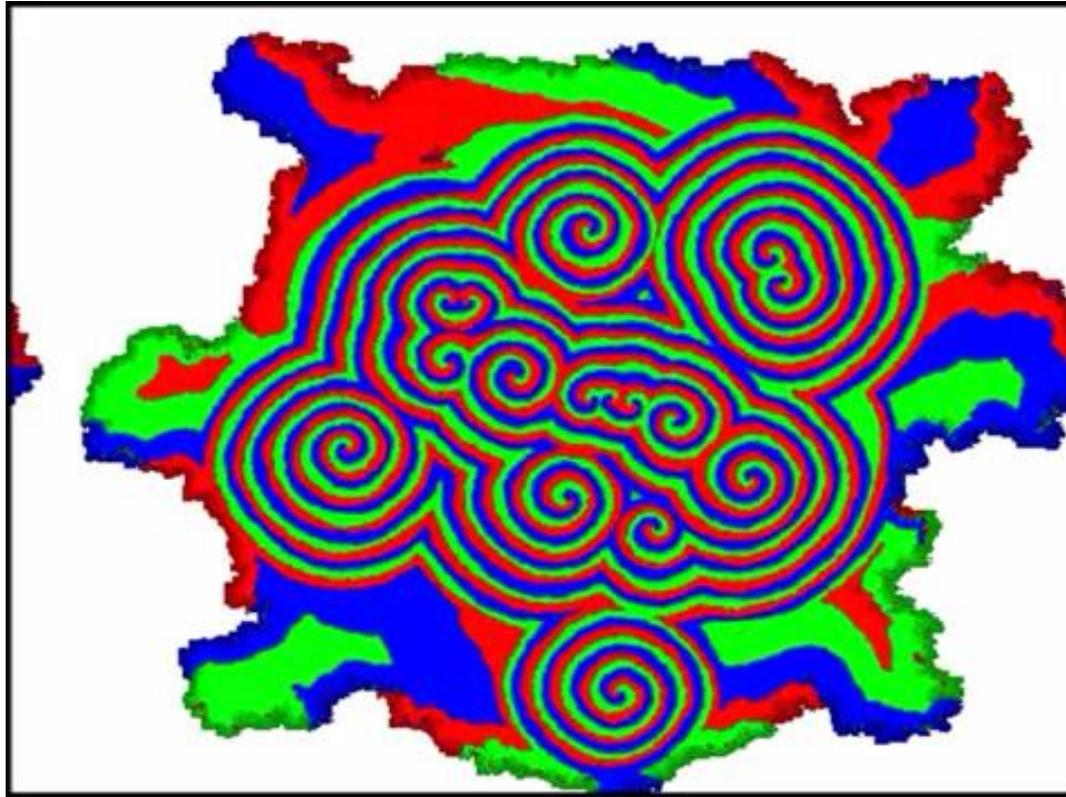
- Model of the **diffusion of autoinducers**: small molecules generated by bacteria as a reaction of the sensed presence of a high-density of other bacteria in the surroundings
- Quorum sensing**: Autoinducers are basic information carriers used by bacteria to take decisions based on the majority, based on the fact that at certain densities certain phenotypical expressions (gene expression) become favorable
- Density is implicitly sensed by the bacteria themselves through the generation of autoinducers, that implements local communication



ROCK-PAPER-SCISSORS AUTOMATA: SIMULATION OF BACTERIAL DIFFUSION (BACTERIAL COMPUTING)

- Three colonies of bacteria (r, p, s) on a lattice
- At each cell: at most one bacteria and one autoinducer molecule
- Bacteria emit light of a specific frequency that depends on the colony
- At each time-step, one bacteria in the grid is randomly selected to perform an *event* with some probability:
 - Reproduction (if there's an empty cell in the neighborhood)
 - Conjugation (transmission of DNA strands between donor and receiver, that needs donor and receiver being in the neighborhood)
 - Autoinducer transmission
 - Each colony emits a different autoinducer
- *Autoinducer molecules act as regulators of the emission of light* from the bacteria according to a **rock-paper-scissor game**:
 - High density of autoinducers from bacteria s represses light emission in bacteria p (i.e., p 's do not express their light emission gene in the presence of a local high density of bacteria s)
 - High density of autoinducers from p represses r 's light emission
 - High density of autoinducers from r represses s 's light emission

ROCK-PAPER-SCISSORS AUTOMATA: A SIMULATION OF BACTERIAL DIFFUSION (BACTERIAL COMPUTING)



<https://www.youtube.com/watch?v=M4cV0nCIzoc>

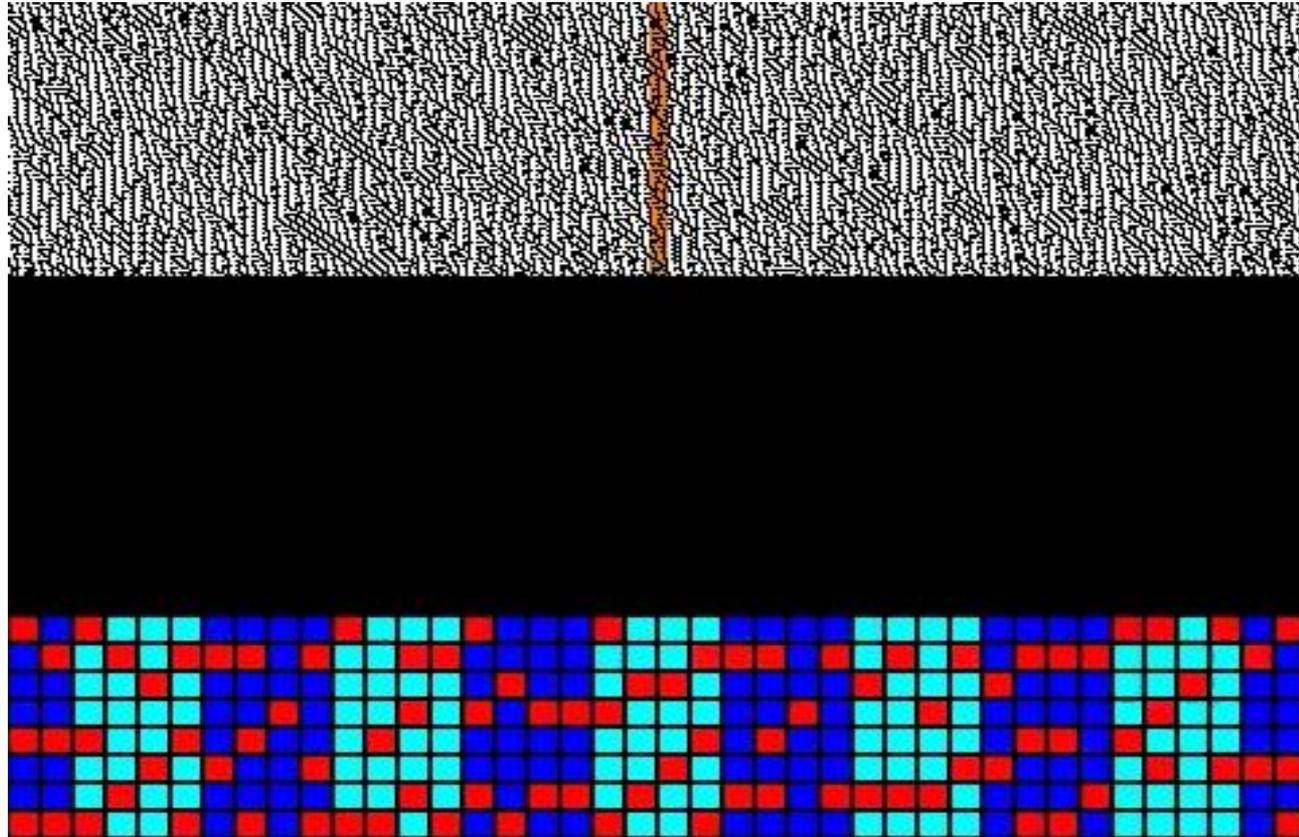
P. Esteba, A. Rodriguez-Paton, *Simulating a Rock-Scissors-Paper Bacterial Game with a discrete Cellular Automaton*, Proc. of IWINAC, LNCS 6687, 2011,

SIMPLE FLUIDS SIMULATION



<https://www.youtube.com/watch?v=9gh6U84KdjA>

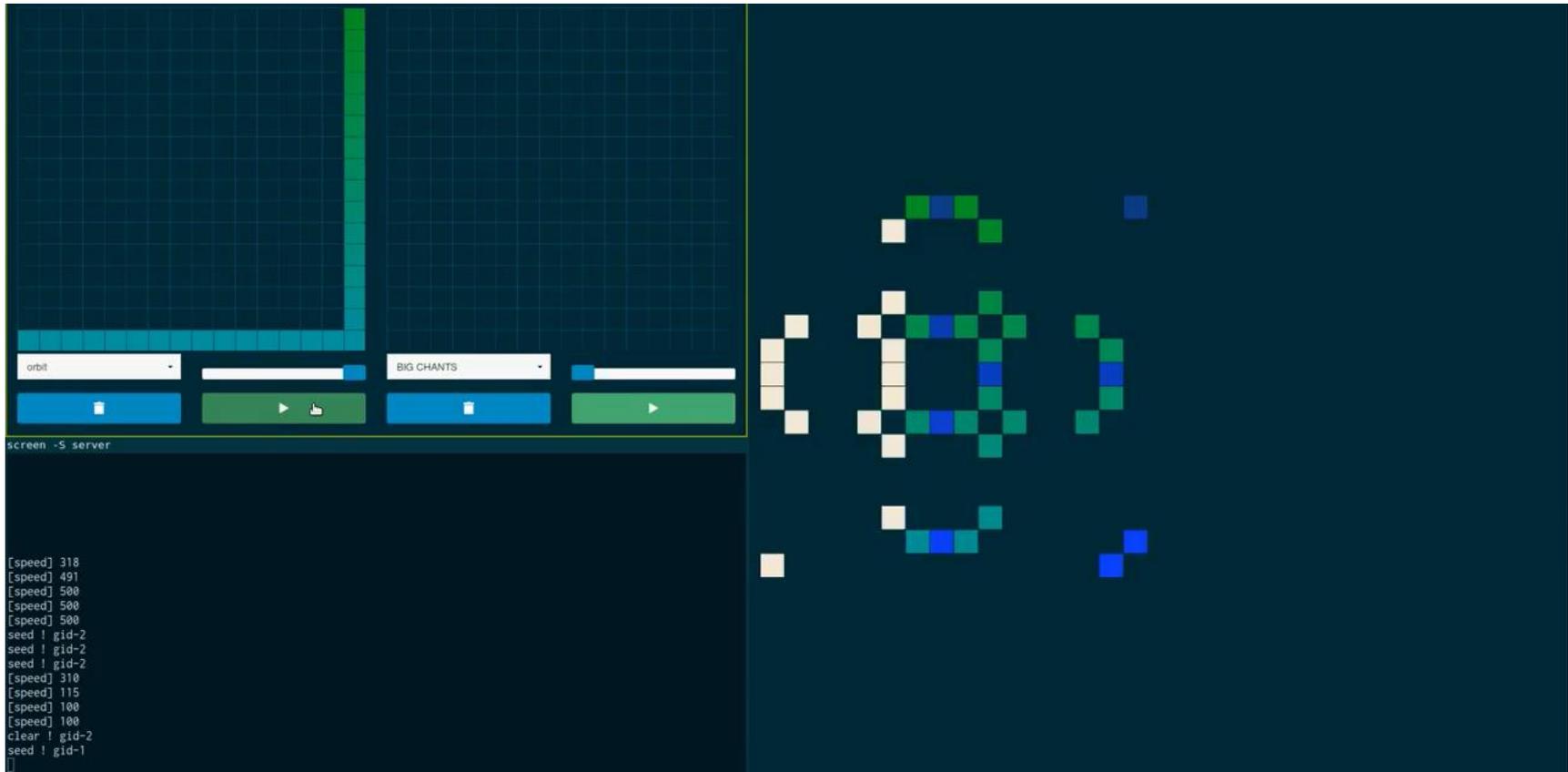
GENERATIVE MUSIC



<https://www.youtube.com/watch?v=ZZu5LQ56T18&t=51s>

D. Burraston, E. Edmonds, *Cellular automata in generative electronic music and sonic art: a historical and technical review*, Digital Creativity, Vol. 16, No. 3, pp. 165–185, 2005

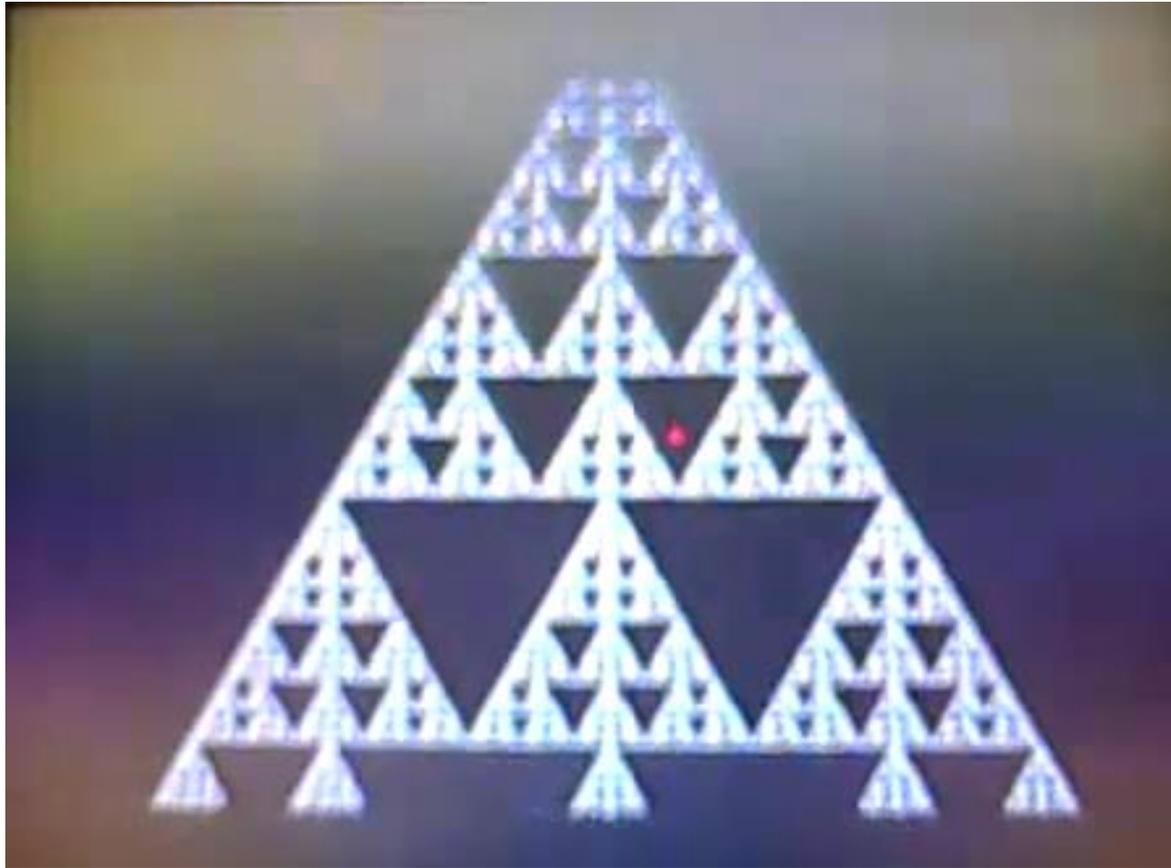
ANOTHER (COOL) WAY OF GENERATIVE MUSIC



<https://www.youtube.com/watch?v=iMvsA8fkVvA&t=84s>

<https://vimeo.com/931182>

CRAZY FRACTAL SOUND



<https://www.youtube.com/watch?v=Dh9EglZJvZs>