Homework 3

Cellular Automata

(Max useful score: 100 - Available points: 190)

15-382: Collective Intelligence (Spring 2019)

Instructions

Homework Policy

Homework is due on Autolab by the posted deadline. As a general rule, you have a total of 6 late days. For this homework you cannot use more than 1 late day. No credit will be given for homework submitted after the late day. After your 6 late days have been used you will receive 20% off for each additional day late.

If you find a solution in any source other than the material provided, you must mention the source.

Submission

Create a zipped archive including: a PDF file with the answers to the provided questions (they can be hand-written, but in this case you must have / use a "readable" handwriting), files that have been used for dealing with the questions that require programming, a README file that specifies how to use / run the programming files. The zipped archive should be submitted to Homework 3 on Autolab.

Contents

1 General properties of Cellular Automata (27 points)

1. Let's consider a two-dimensional, binary CA with 100 cells. The neighborhood is a Moore one (each cell has 8 neighbors). Boundary conditions are periodic.

   (a) (3 points) How many different configurations (states) can be potentially achieved by the CA?

   (b) (3 points) How many evolution functions (i.e., Boolean functions) can be defined for such a CA?

(c) (3 points) If we know that the CA features m configurations that are *Garden of Eden*, what is an upper bound on the number of different configurations that can be visited by the CA starting from an arbitrary configuration?

2. (4 points) Let's consider the following tree construction. Node 0 is at the root and has 3 children, nodes 1,2,3. From this first level on, each node has two new children, and the process is repeated until some level n is reached. Can the resulting structure be used as a lattice for the definition of a CA? Justify your answer, explaining why yes/no the structure defines a lattice (or under which conditions it does).

3. If you have answered 'yes' to the previous question:
   (a) (3 points) How many nodes (cells) would be in the CA for $n = 4$? (provide a formula for the computation of the number of cells for a generic n).
   (b) (3 points) How would you define three different types of neighborhoods for the CA?
   (c) (2 points) How would you define boundary conditions?

4. (4 points) What are the defining properties of the CA used in the model of Nagel-Schreckenberg for traffic simulation? (You should briefly explain if the state of the cell is binary or not, what are the boundary conditions, what is the neighborhood, synchronous or asynchronous updating, etc.)

5. (2 points) What does it mean *rule 184* in Wolfram's classification of elementary CA?

2 Behavior classes for elementary CA (33 points)

For each one of the following CA:

1. Rule 4
2. Rule 22
3. Rule 30
4. Rule 54
5. Rule 90
6. Rule 110
7. Rule 170
8. Rule 250

- Describe to which class it belongs to and why, based on observing a few sample runs. At this aim you can/should use the python code (class `CellularAutomata`) provided with the homework.
- Extend the provided python code to compute Lyapounov exponents and relate the observed values of the Lyapounov exponents to the class properties (e.g., if the CA is of class IV, a chaotic behavior should be observed, that in turn should be reflected in the Lyapounov exponents, while if it is of class one or two the propagation of information should follow a different pattern). Report and discuss the results.

3 Traffic studies using Cellular Automata (65 points)

You work for the traffic department and have to make an analysis of the traffic flows along one major busy road that goes through the capital city. The purpose of the analysis is to understand how fluid or jammed / irregular traffic flows are, and then, possibly, devise some corrective measures. For the analysis you decide to use a microscopic traffic model based on a cellular automata according to the Nagel-Schreckenberg + DVR model and Rickert-Nagel-Schreckenberg model (as described in the lecture slides and in the original research papers provided on the course website).

The road is two-lanes one-way, has in and out ramps, and traffic lights. here are the parameters describing the scenario:

- Length $L = 2.7$ Km

- A car is 7.5m long
- $v_{max} = 5$/cells per time-step
- Periodic boundary conditions
- Inflow ramp at 0.75 Km, probability of a car entering is p_i, subject to the space in the road
- Outflow ramp at 2.4 Km, probability of a car exiting is p_o subject to the car having $v > 0$
- Three traffic lights, s_1, s_2, s_3, at the following positions (in Km): 0.75, 1.5, 2.1. Each traffic signal has a periodic timing for switching between green and red light status, that we indicate, respectively, with τ_1, τ_2, τ_3, (e.g., s_1 changes status from green/red to red/green every τ_1 time steps).

Let's set $\tau_1 = \tau_2 = \tau_3 = 5$. The average density ρ (number of cars per cell) of vehicles traveling in the road is constant, with $p_i = 0.3$. The randomization parameter p that models human (mis)behavior in the Nagel-Schreckenberg model is set to 0.5. The slow-to-start randomization parameter is set to $p_0 = 0.75$.

3.1 Design and implement the agent-based CA model (30 points)

Implement a CA that models the scenario as described. Set all missing parameters according to some "reasonable" guess. Run simulations that correspond to 3h of real time.

Implement a simple visualization of the time evolution of the CA cells in the road lanes, as it is normal practice to visualize a CA (i.e., a new CA cell line per time-step)

3.2 Graphical visualization (10 points)

In order to appreciate (and better understand, for you and for me) what's going on, implement a graphical visualization of the traffic along the road, that should be kept fixed (i.e., show how the flows proceed over time along the road).

3.3 Analysis of the observed behaviors (25 points)

Make an analysis of what happens, reporting the plots of:

- Flux vs. the density $\rho \in [0.05, 1]$;
- Average speed vs. density;
- Variance of the speed vs. density;
- Average length of traffic jams vs. density;
- Spatial distribution of traffic jams for the critical density at which the flux starts decreasing.
- Start with an empty road $\rho = 0$, set all signals to constant green, close the outflow ramp. Set the $p = 0.015$, $p_0 = 0.75$, and $p_i = 0.4$. As time passes, the density will increases steadily because of the inflow. Run the simulation until $\rho = 1$. Show flux vs. density.

Thoroughly discuss the results that you observe for each one of the above plots. Point out any interesting behaviors you might observe, such as phase transitions, metastability, ...

Make possible "suggestions" that the traffic department could use for improving the traffic situation.

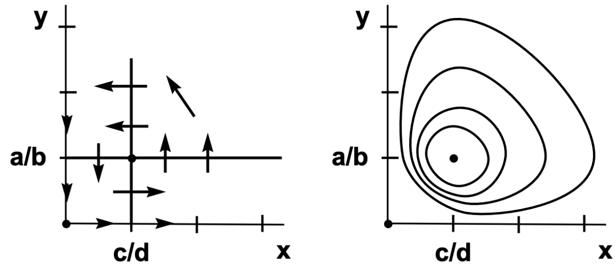
4 Cellular Automata as an agent model for studying interacting populations (65 points)

Using the formalism of dynamical systems, we have studied a few models of competing populations / species. Among these models, the Lotka-Volterra model that considers a basic prey-predator scenario is a remarkable one.[1] You can find a compact yet informative description of the model and of the related dynamics in the

[1] The model was developed independently by Alfred Lotka and by Vito Volterra around 1925. It has been the basis of a large number of studies and extensions of the basic model.

book extract provided in the course website for the lecture of February 3, 2019. Here the equations for the prey-predator scenario are summarized together with a plot of the phase space, that shows the cycling behavior of the two population sizes, as well as the presence of a center as a stability point.

$$\begin{cases} \dot{x} = ax - bxy \\ \dot{y} = -cy + dxy \end{cases}$$



The Lotka-Volterra model is a form of a population model based on the *well-mixing* (or homogeneous mixing) hypothesis: the rate of encounter between members of two different populations is proportional to the product of the population sizes. The well-mixing hypothesis implies that doubling the size of either population results in twice as many encounters, and this implies that members of both populations are *homogeneously* distributed in space and do not mix in any smaller subgroups. In practice, the well-mixing assumption it allows the use of ODEs instead of PDEs (or of agent-based models), that would otherwise require the introduction (and modeling) of the spatial structure (including, for instance, mobility).

The well-mixing hypothesis is a form of *mean field* assumption, since an *average* effect is assumed when considering multiple individuals interacting to each other. In other words, in the case of two populations N_1, and N_2, instead of considering each possible location for an individual and one-to-one interactions, a mean (force / interaction) field is considered, reducing in this way the $N_1 \times N_2$ multi-agent problem to a two-agent one. Each population is reduced to one single entity that has everywhere the same mean value (in relation to the properties of interaction). This approximation works well for large populations, since they tend to be uniformly distributed, and local fluctuations become less important. However, it is clear that when one of the two populations decreases because of the competition, the mean field approximation can become significantly wrong because the assumption of uniformity (in space) is hardly met. Also when space has a well defined structure (that in turn affects interaction), the mean field assumption might not work well.

In order to possibly overcome the limits of the mean field approximation, let's define a Cellular Automata model, that can bring a few advantages: a spatial structure (defined in terms of a lattice model), (more complex) agent-based interactions, probabilistic components.

4.1 CA model for prey-predator systems (25 points)

Define a Cellular Automata model for a prey-predator system. The CA must model a relatively large rectangular territory (with periodic boundaries) where an initial population of P preys and K predators are present (scattered around). The following aspects of the evolution of each population and their mutual interaction must be accounted for in the state transition function for CA's cells.

- The same mechanisms that are modeled by the Lotka-Volterra equations should found a counterpart in the CA model: exponential growth rate (birth) rate of the prey population in the absence of predators; exponential decrease (mortality by starving) in the predator population in the absence of prey; mortality of preys as the result of encounters with predators; increase of the predator population as a result of encounters with preys.
- *Mobility* of preys and predators. Mobility must happen respecting adjacency relationships between cells (i.e., no "jumps"). Mobility must be driven by a purpose (e.g., escaping from preys, escaping from crowded areas, going to where grass is present).

It can be observed that the elements above potentially provide a more detailed (realistic) model of a prey-predator scenario compared to the Lotka-Volterra equations.

You must precisely specify ALL the elements that define a Cellular Automata model: states, neighborhoods, transition functions, lattice, updating mechanisms.

You can (should) use *probabilistic rules*. You can also consider to subdivide each updating step in sub-steps (e.g., first interaction-reproduction rules, then mobility rules).

Once you have built the model and implemented the CA, you are asked to perform a simulation study of the behaviors resulting from the model, as it is specified below. The simulation study must be supported by some

2D *visualization* showing the status of a cell over the epochs (e.g., check for instance the visualization in the wolf-sheep model in NetLogo https://ccl.northwestern.edu/netlogo/).

4.2 Simulation study of the CA model (20 points)

Consider a 100×100 grid lattice. If your rules are "well" designed, by starting with different initial populations (of prey and predators), you should be able to observe an exponential growth of preys in absence of predators, and an exponential decay of predators in absence of preys. Moreover, you should observe (more or less) the cycling behaviors that arise in the phase space of the Lotka-Volterra equations, with the cyclic evolution of the preys always lagging behind that of the predators.

Make plots with the above results and discuss the impact of the different parameters that you have used. That is, in order to achieve the goal of producing results comparable to Lotka-Volterra equations, you might need to adjust parameters, neighborhoods, transitions, etc. Therefore, you are asked to comment on the different choices that you have made and why do they work or didn't work as expected.

4.3 Extended prey-predator model including preys' food (10 points)

In order to complete the food cycle in the ecosystem, a third of type of agent must be included in the system: the *food* F for preys (e.g., grass). F is expected to have an exponential growth rate in the absence of preys (consumers). In the absence of food, the preys' birth rate gets dramatically decreased (this models "crowded" scenarios: if many preys are in a region that does not provide enough food they will starve and die).

Consider the full model, including F. How does this change the dynamics? Are the observations reasonable from a real-world point of view?

4.4 Adding a shelter for the preys (10 points)

Introduce an additional spatial component to the model: a *shelter* S, of 5×5 cells. This is an area to where predators cannot enter. However, the shelter doesn't have food, such that being too long in the shelter makes the preys starving and then eventually die.

Again, perform a simulation study and discuss the observed behaviors.