Lecture 25:
Swarm Intelligence 6 / Ant Colony Optimization 2

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SHORTEST PATHS WITH PHEROMONE LAYING-FOLLOWING

Pheromone Intensity Scale

#Pheromone on a branch ∝ Frequency of fw/bw crossing ∝ Length (quality) of paths
Let’s abstract a more complex scenario.

- **Multiple decision nodes**: $n$ decision states/nodes, $x_1, x_2, ..., x_n \in X$

- Set $A$ of decisions/actions, $a_1, a_2, ... a_m$, such that at each state $x$ a subset $A(x)$ of actions is available or feasible

- A **path (ant solution)** is constructed through a sequence decisions, for each visited state

- **Multiple ants iterating path construction** (i.e., foraging) in parallel

- A **traveling cost** is associated to each state transition: colony’s goal is to let the ants moving over the **minimum-cost path between nest and food**
Let’s abstract a more complex scenario

- **Distributed Optimization Problem**
- At each state $x_k$ only local information / constraints (+ some ant memory) is available for taking (a possibly optimized) decision $a \in \mathcal{A}(x_k)$
  - Pheromone information (dynamic), parametrized as a vector $\tau^k$ (stigmergic variables)
  - Heuristic information (static, scenario-related) parametrized as a vector $\eta^k$
- Ant behavior: **Stochastic decision policy** $\pi_\varepsilon(x_k; \tau^k, \eta^k)$, $\pi_\varepsilon: X \mapsto A$

**How ant colonies solve the Distributed MCP problem?**

Exploiting pheromone for learning the best (parameters) of the decision policy
ANT COLONIES: INGREDIENTS FOR SHORTEST PATHS

- A number of concurrent autonomous (simple?) agents (ants)
- Forward-backward constructive **path sampling** based on the stochastic policy $\pi_\varepsilon$
- Local laying and sensing of pheromone $\rightarrow$ **Pheromone is dynamically updated**
- Step-by-step stochastic decisions biased by local pheromone intensity and by other local heuristic aspects (e.g., terrain)
- Multiple paths are concurrently tried out and implicitly evaluated
- **Positive feedback effect** (local reinforcement of good decisions)
- Iteration over time of the path sampling actions
- **Persistence** (exploitation) and evaporation (exploration) of pheromone
• Let's mimic ant colonies, with some pragmatic modifications ...  

• Once completed a solution / path:
  • The sampled solution is **evaluated** (e.g., sum of the individual costs)  
  • “Credit” is assigned to each individual decision belonging to the solution  
  • **Pheromone updating**: the value of the pheromone variables $\tau^k$ associated to each decision in the solution are **modified** according to the “credit”  
  • Pheromone values can also **decade/change** for other reasons (e.g., **evaporation**)  
  • **Pheromone values locally encode how good is to take decision i vs. j** as collectively estimated/learned by the agent population through repeated solution sampling
ANT COLONY OPTIMIZATION METAHEURISTIC:
(VERY) GENERAL ARCHITECTURE

procedure ACO_metaheuristic()
  while (¬ stopping criterion)
    schedule activities
    ant_agents.construct.solutions.using.pheromone();
    pheromone.update();
    daemon.actions(); /* optional */
  end schedule activities
end while
return best.solution.generated;

• Solution construction
• Monte Carlo path sampling by \( N \) (# states) joint probability distributions parametrized by \( \tau \) and \( \eta \) variable arrays
• Sequential learning by Generalized Policy Iteration (GPI)
At each decision node $i$ an array of pheromone variables: $\tau_i = [\tau_{ij}] \in \mathbb{R}$, $\forall j \in \mathcal{N}(i)$ is available to the ant agent to issue the routing decision.

$\tau_{ij} = q(j|i)$: estimate of the quality of moving to next node $j$ conditionally to the fact of being in $i$. Pheromone values are collectively learned by the ants through path sampling.

At each decision node $i$ an array of heuristics variables $\eta_i = [\eta_{ij}] \in \mathbb{R}$, $\forall j \in \mathcal{N}(i)$ is also available to the ant agent to take the routing decision.

$\eta_{ij}$ is also an estimate of $q(j|i)$ but it is derived from a process or a priori knowledge not related to the ant actions (e.g., node-to-node distance).
Given $G(V, E)$ find the Hamiltonian tour of minimal cost: NP-Hard

Every cyclic permutation of $n$ integers is a feasible solution

$\pi_1 = (1, 3, 4, 2, 6, 5, 7, 1), \quad \pi_2 = (2, 3, 4, 5, 6, 7, 1, 2)$

$c(\pi_2) = d_{23} + d_{34} + d_{45} + d_{56} + d_{67} + d_{71} + d_{12} = 93$

Read also as set of edges:
{(2,3), (3,4), (4,5), (6,7), (7,1), (1,2)}

It’s easier to consider fully connected graphs, $|E| = |V| |V-1|$: If two nodes are not connect, $d$ is infinite

“Related” combinatorial optimization problems: VRPs, SOP, TO, QAP, …
ACO FOR THE TRAVELING SALESMAN PROBLEM (TSP)

- **Pheromone variables**: $\tau_{ij} \in \mathbb{R}^+$ expresses how beneficial is (estimated, up to now) to have edge $(i,j)$ in the solution to optimize final tour length $\rightarrow |E|$ variables

- **Heuristic values** $\eta_{ij} \in \mathbb{R}^+$: problem costs $c_{ij} \in \mathbb{R}^+$ for traveling from $i$ to $j$ $\rightarrow |E|$ variables

**Solution construction strategies** (no repair, no look-ahead)

- **Extension**: when ant $k$ is in city $i$, how good is expected to include (feasible) city $j$ (next in the solution sequence $x^k(t)$)? $\rightarrow f(\tau_{ij}, \eta_{ij})$

- **Insertion**: how good is expected to insert (feasible) edge $(m,p)$ in the partial solution $x^k(t)$? $\rightarrow f(\tau_{mp}, \eta_{mp})$
Initialize $\tau_{ij}(0)$ to small random values and let $t = 0$;

repeat

Place $n_k$ ants on randomly chosen origin nodes;

foreach ant $k = 1, \ldots, n_k$ do

Construct a path $x^k(t)$ [Update pheromone step-by-step];
Evaluate path $x^k(t)$;
end

foreach [selected] edge $(i, j)$ of the graph do

Pheromone evaporation;
end

foreach [selected] ant $k = 1, \ldots, n_k$ do

foreach [selected] edge $(i, j)$ of $x^k(t)$ do

Update $\tau_{ij}$ using path evaluation results;
end
end

Daemon actions [Local search];

$t = t + 1$;

until stopping condition is true;

return best solution generated;