LECTURE 30: TASK ALLOCATION 3

TEACHER:
GIANNI A. DI CARO
Solution Approaches

- Use the reference optimization models in a centralized scheme, solving the problems to optimality (e.g., Hungarian algorithm, IP solvers using branch-and-bound, optimization heuristics)

- Use the reference optimization models adopting a top-down decentralized scheme (e.g., all robots employ the same optimization model, and rely on local information exchange to build the model)

- Adopt different solution models avoiding to explicitly formulate optimization problems.

- Market-based approaches are an effective and popular option

- Emergent/Swarm approaches: effective / simpler alternative
Basic Ideas of Emergent TA

Ideas and models from clustering and labor division behaviors in ant colonies

Brood care:
- Larvae are sorted in such a way that different brood stage are arranged in concentric rings
- Smaller larvae are in the center, larger larvae on the periphery

Cemetery organization:
- Clustering corpses to form cemeteries
- Each ants seems to move randomly while picking up or depositing (dropping) corpses
- Pick up or drop: decision based on local information
- The combination of these very simple behaviors from individual ants give raise to the emergence of colony-level complex behaviors of cluster formation
Task Allocation Based on Response Threshold

- **Response thresholds** refer to the likelihood of reacting to task-associated stimuli (e.g. the presence of a corps or a larva, the height of a pile of dirty dishes to wash)

- Individuals with a low **threshold** perform a task at a lower level of stimulus than individuals with high thresholds

- **Individuals become engaged in a specific task when the level of task-associated stimuli exceeds their thresholds**

- If a task is not performed by individuals, the intensity of the corresponding stimulus increases

- Intensity decreases as more ants (agents) perform the task

- **The task-associated stimuli serve as stigmergic variable**
Let $s_j$ be the intensity of task-$j$-associated stimuli

A response threshold, $\theta_{kj}$, determines the tendency of individual $k$ to respond to the stimulus, $s_j$, associated with task $j$

Individual $k$ engages in task $j$ with probability

$$P_{\theta_{kj}}(s_j) = \frac{s_j^\omega}{s_j^\omega + \theta_{kj}^\omega}$$

where $\omega > 1$ determines the steepness of the threshold

For $s_j << \theta_{kj}$, $P_{\theta_{kj}}(s_j)$ is close to zero, and the probability of performing task $j$ is very small

For $s_j >> \theta_{kj}$, the probability of performing task $j$ is close to one
**Single Task Allocation**

- Assume only one task
- The probability that an inactive ant will become active

\[
P(\vartheta_k = 0 \rightarrow \vartheta_k = 1) = \frac{s^2}{s^2 + \theta_k^2}
\]

\(\vartheta_k = 0\) indicates that ant \(k\) is inactive
\(\vartheta_k = 1\) indicates that the ant is performing the task
- An active ant spends an average \(1/p\) time performing the task
- Change in stimulus intensity:

\[
s(t + 1) = s(t) + \sigma - \gamma n_{act}
\]

\(\sigma\) is the increase in demand
\(\gamma\) is the decrease associated with one ant performing the task
\(n_{act}\) is the number of active ants
The more ants engaged in the task, the smaller the intensity, $s$, and consequently, the smaller the probability that an inactive ant will take up the task.

If there are not enough ants busy with the task (i.e. $\sigma > \gamma n_{act}$), the probability increases that inactive ants will participate in the task.

Multiple tasks:
- Let there be $n_j$ tasks
- Let $n_{kj}$ be the number of workers of caste $k$ performing task $j$
- Each individual has a vector of thresholds, $\theta_k$
- After $1/p$ time units of performing task $j$, the ant stops with this task, and selects another task
Market-based: Basic Ideas

- Based on the economic model of a free market
- Each robot seeks to maximize individual “profit”
- Individual profit helps the common good

1. An auctioneer (i.e. a robot spotting a new task) offers tasks (or roles, or resources) in an announcement phase

2. Robots can negotiate and bid for tasks based on their (estimated) utility function

3. Once all bids are received or the deadline has passed, the auction is cleared in the winner determination phase: the auctioneer decides which items to award and to whom.

4. Decisions are made locally but effects approach optimality
   - Preserve advantages of distributed approach
MARKET-BASED: BASIC IDEAS

- Robots model an economy:
  - Accomplish task → Receive revenue
  - Consume resources → Incur cost
  - Robot goal: maximize own profit
  - Trade tasks and resources over the market → Auctions

- By maximizing individual profits, team finds a globally good solution

- Time permitting → More centralized

- Limited computational resources → More distributed
Utility = Revenue − Cost

Team revenue = Sum of individual revenues

Team cost = Sum of individual costs

Costs and revenues are set up per application

- Maximizing individual profits must move team towards globally optimal solution

Robots that produce well at low cost receive a larger share of the overall profit
MARKET-BASED: IMPLEMENTATIONS

- M+ (Botelho and Alami, ICRA 1999)
- TraderBots (Dias et al., multiple publications 1999-2006)
Auctions are an effective and practical approach to task allocation, and more in general to agent-coordination.

Auctions have a small runtime.

Auctions are communication efficient:
  - Information is compressed into bids.

Auctions are computation efficient:
  - Bids are calculated in parallel.

Auctions result in a small team cost.

Auctions can be effectively used in dynamic problem environments.
**Auctions**

- **Definition** [McAfee & McMillan, JEL 1987]: a market institution *with an explicit set of rules* determining resource allocation and prices on the basis of *bids* from the market participants.
**Auctions**

- **Definition** [McAfee & McMillan, JEL 1987]: a market institution *with an explicit set of rules* determining resource allocation and prices on the basis of **bids** from the market participants.

- Used since ever (500 B.C. in Babylon, women for marriage) and for many **commodities**:

  Antiques and art, Livestock and other agricultural produce, Real estate, Mineral and timber rights, Radio frequencies, Diamonds, Corporate stock, Treasury bonds, Used automobiles, Wives and slaves, Body parts and human tissues ...
**Motivation: Attributing the Right Price**

**Pricing models:**

- **Posted price**
  - Static
  - Dynamic:
    - Change dynamically over time
    - Customized pricing

- **Price discovery mechanisms:**
  - Negotiations
  - Auctions

In the *economy*
Why Auctions?

- For objects of unclear value
- Mechanized:
  - Reduces the complexity of negotiations
  - Ideal for computer implementation
- Creates a sense of “fairness” in allocation when demands exceed supply
FORMATS

(Foward) Auction

Reverse Auction

Double Auction Exchange

Increasing prices

Decreasing prices
Design Space

Auction Format
- Open vs. closed
- Ascending vs. descending
- Simultaneous vs. sequential
- Single vs. multi-round

Participation Rules
- Participant requirements
- Preferred bidding status
- Fees

Information
- Goods/services
- Bids
- Participants
- Transaction history

Bidding Rules
- Price-quantity schedules
- Bid components
- Bundle, Combinatorial
- Activity rules

Clearing
- Winner determination or matching
- Who pays and how much?
- Clear timing

I will choose the rules of the game.
OK, but choose them well!

Mechanism Designer
Players
Bidding Strategies

- **At which auctions** to participate?
  - Participation cost, auction duration, number of bidders
- **When** to bid?
- **How much** to bid? (price and/or quantity)
  - Effects of synergies or economies of scale
Auctions with Human Participants

- **Efficient allocation**: the bidders who values an item most gets it
  - Incentives for *truthful bidding*
- Maximize the *auctioneer’s revenue*
- *Things to avoid:*
  - **Collusion**: If some bidders collude, they might do better by *lying*
  - Collusion among buyers, sellers, and/or auctioneer.
  - *Hide-in-the-grass strategy*
  - Predatory bidding
  - Jump bidding
  - Shilling
  - Bid shielding
  - Winner’s
Auctions with Robots

- Robots *don’t game the system*, e.g. by bidding untruthfully.
  - They bid as we design them to!
- Robots do not intentionally hide information and thus do **not** have privacy concerns.
- Robots **do not have inherent utilities** (preferences).
  - *We define their utilities* so that the result of the auction serves a common *team objective*.
- Robots don’t care if the outcome is not “fair.”
Auctions Mechanisms

- **Open-cry vs. Sealed bid →** Different information accessible, online vs. offline
- **Reserve prices**

For Task Allocation:

- **Single-item** auctions
- **Multi-item** auctions
- **Combinatorial** auctions
Single Item Auctions

- Auctioneer is selling a **single task**
- **First-price auction**
- **Protocol:** Each bidder submits a bid containing a single number representing its cost for the task. The bidder with the **lowest bid wins** and is awarded the task, agreeing to perform it for the price of its bid.

![First-Price Auction Diagram](image)

  - Bidders’ rational strategy is to bid the **smallest price that the bidder is willing to pay and that will secure the good.**
  - *If* a bidder knew all other bidders’ valuations of the good, this smallest price would equal the highest of others’ valuations (plus slightly more): **the 2nd price**
The seller doesn’t really maximize its profit since the item is sold not to the highest value the bidder that value it the most would pay, but only to a value which is slightly higher than the bid of the second highest bidder.
**Single Item Auctions**

- **Vickrey (second-price) auction**
  - Protocol: Same as first-price above, but bidder with the lowest bid agrees to perform task for the price of the second-lowest bidder’s bid
- **Incentive compatible**

**Second-Price Auction**

- 200$ (sold to purple gent.)
- 100$
- 50$
- 150$
- 250$

**Which mechanism for robots?**

- Doesn’t matter if robots bid truthfully
Multi-Item Auctions

- **Protocol:** Auctioneer offers a set of $T$ tasks. Each bidder may submit bids on some/all of the tasks. The auctioneer awards one or more tasks to bidders, *with at most one task* awarded to each bidder
  - No multiple awards: bids do not consider cost dependencies

- Protocol may specify a fixed number of $m$ awards out of the $T$ tasks:
  1. $m$ tasks awarded, $1 \leq m \leq \#bidders$
  2. Every bidder awarded exactly one task ($m = \#bidders$)
  3. The one best award ($m = 1$)

- For 2. the assignment can be done optimally [Gerkey and Matarić, 2004]
  - Greedy algorithm: Award the lowest bidder with the associated task, eliminate that bidder and task from contention, and repeat until you run out of tasks or bidders.
Protocol: Auctioneer offers a set of tasks $T$. Each bidder may submit bids on any task bundles (subsets of $T$), and the auctioneer awards a combination of bundles to multiple bidders (at most one bundle awarded per bidder). The awards should maximize the revenue for the auctioneer.

- Exponential number of bundles, $2^{|T|}$
  - Winner determination is **NP-hard**
  - But, fast optimal winner determination algorithms exist that take advantage of the sparseness of the bid set [e.g. CABOB, Sandholm 2002]

- Number of bundles can be reduced
  - **Auctioneer**: only allow certain bundles
    - Roles [Hunsberger and Grosz 00]
    - Rings or nested structure [Rothkopf et al. 98]
  - **Bidders**: task clustering algorithms [Berhault et al. 03, Nair et al. 02]
    - Clustering (spanning tree, greedy nearest neighbor)
    - Limit bundle size
    - Recursive max graph cuts
Types of Auctions for Task Allocation

- Parallel Auctions
  - Each robot bids *on each task* in independent and simultaneous auctions

- Combinatorial Auctions
  - Each robot bids *on some bundles* (= subsets) of tasks

- Sequential Auctions
  - There are *several parallel auctions bidding rounds* until all tasks have been assigned to robots. Only one task is assigned in each round. A bundle is assigned at the end of the rounds