15-382 Collective Intelligence – S19

Lecture 32:
Task Allocation 5

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Sequential Auctions: Bidding Rules

- Robots' bids should be quantified to let a robot win a task so that some selected measure of ***team performance is optimized***

- Let's assume that team performance is in terms of costs minimization
  → A robot's bid should be related to the increase of some measure of team cost, such that the best bid increases the cost (of the team) least

- Robot r bids on task t the difference in the minimal measure of the team cost for the given team objective between the allocation of targets to all robots that results from the current allocation if robot r wins target t and the one of the current allocation. (Targets not yet won by robots are ignored.)

- Team cost minimization can be achieved in a fully distributed way: if each robot bids to minimize **its own** cost difference, it actually minimizes the cost difference for the team

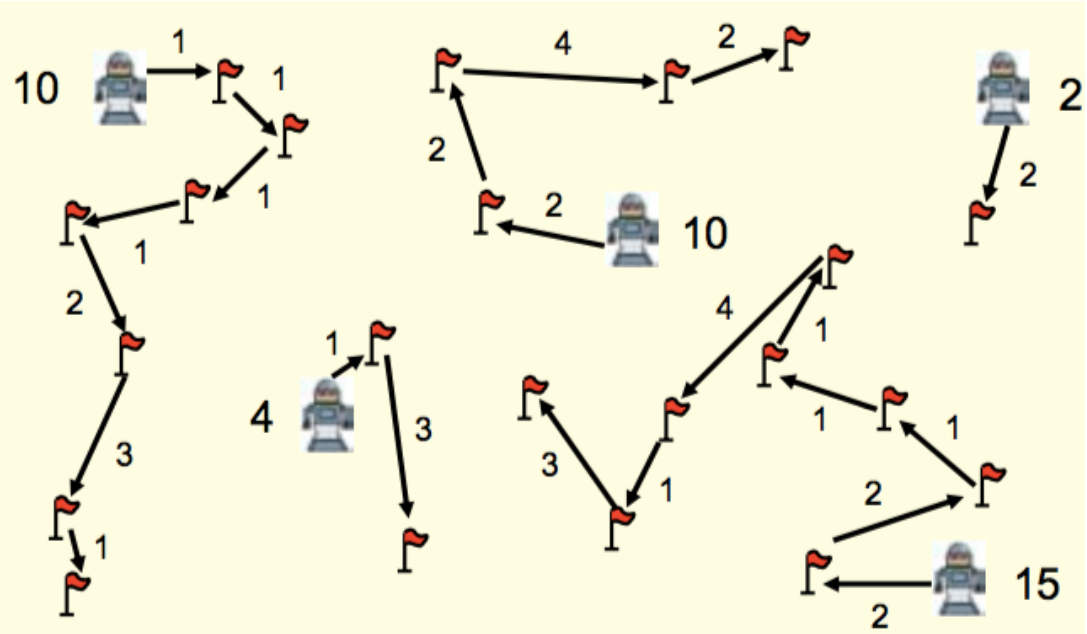
Team Performance Criteria

- **MiniSum**
  - *Team goal:* Minimize the sum of the path costs over all robots
  - → Minimization of total used *energy* or traveled *distance*
  - Application: logistics / goods delivery, planetary surface exploration
- **MiniMax**
  - *Team goal:* Minimize the maximum path cost over all robots
  - → Minimization of total completion time (*makespan*)
  - Application: facility surveillance, mine clearing, time-critical task set
- **MiniAvg**
  - *Team goal:* Minimize the average arrival time over all targets
  - → Minimization of average service time (*flowtime*)
  - Application: search and rescue

MiniSum and Bidding Rules in Seq Auctions

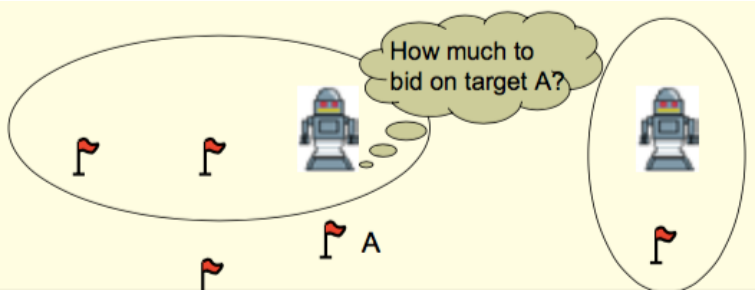
- **MiniSum**
  - *Team Goal:* Minimize the sum of the path costs over all robots
  - → Minimization of total used *energy* or traveled *distance*
  - Application: logistics / goods delivery, planetary surface exploration

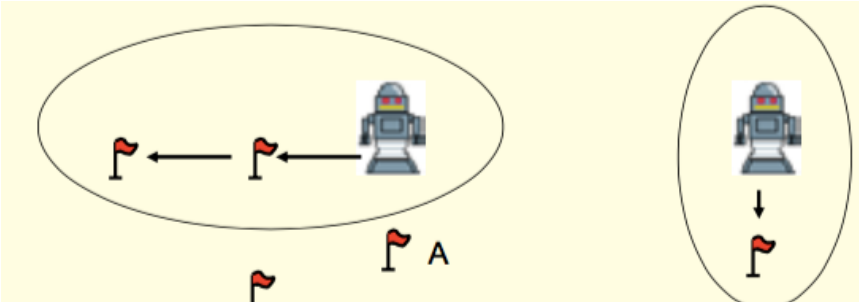
E.g., Team performance: 10+10+2+4+15 = 41



MiniSum and Bidding Rules in SeQ Auctions

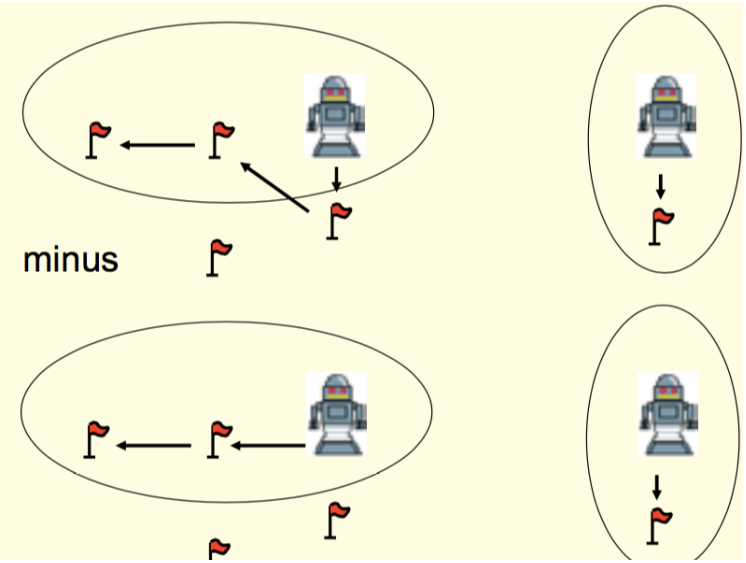
- *Bids ↔ Team Goal:* Minimize the sum of the path costs over all robots





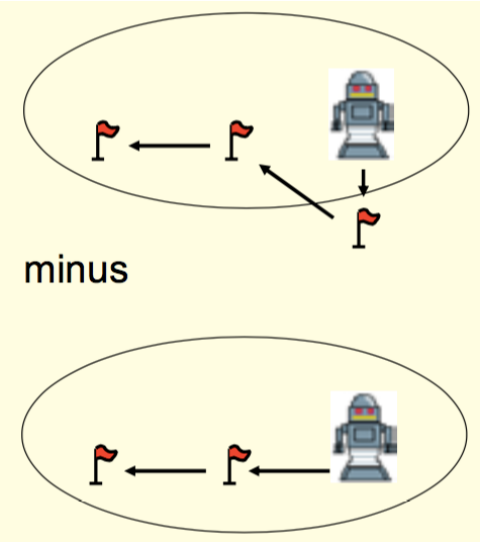
Current allocations and costs

Bid that increases the team cost the least



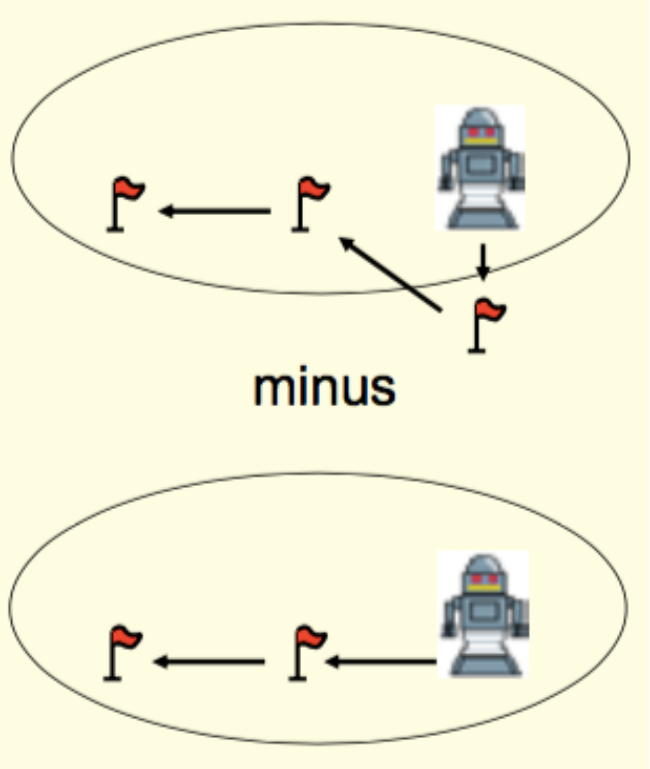
Equivalent to

A robot doesn't need to know about other robots



MiniSum and Bidding Rules in Seq Auctions

- *Bids* ↔ *Team Goal:* Minimize the sum of the path costs over all robots



- minimal path cost the robot needs from its current location to visit all targets it has won if it wins the target that it bids on

minus

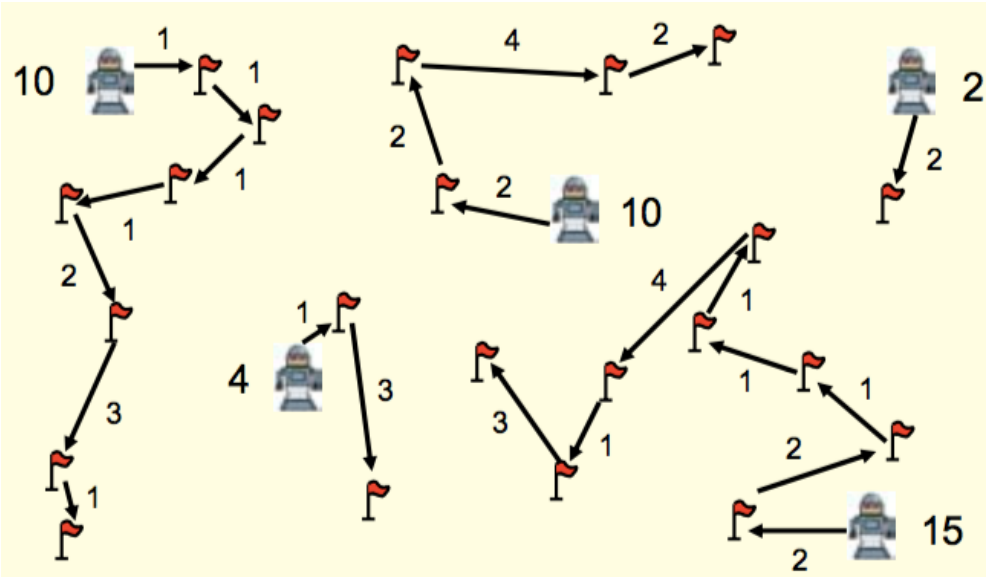
- minimal path cost the robot needs from its current location to visit all targets it has won so far

Bid the increase in the minimal path cost the robot needs from its current location to visit all targets it has won if it wins the target it is bidding on (***BidSumPath***)

MiniMax and Bidding Rules in Seq Auctions

- **MiniMax**
  - *Team goal:* Minimize the maximum path cost over all robots
  - → Minimization of total completion time (*makespan*)
  - Application: facility surveillance, mine clearing, time-critical task set

E.g., Team performance: $\max(10, 10, 2, 4, 15) = 15$

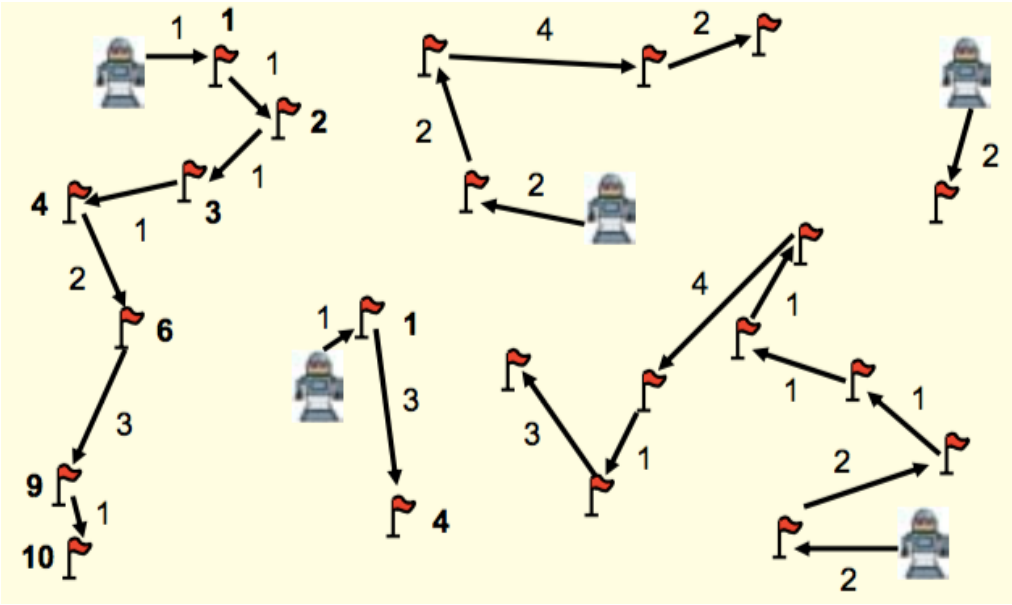


- **Bid** the minimal path cost the robot needs from its current location to visit all targets it has won *if* it wins the target it is bidding on (***BidMaxPath***)
- This bid automatically balances the path costs of all team robots, without the need for knowing about their bids

MiniAvg and Bidding Rules in Seq Auctions

- **MiniAvg**
  - *Team goal:* Minimize the average arrival time over all targets
  - → Minimization of average service time (*flowtime*)
  - Application: search and rescue

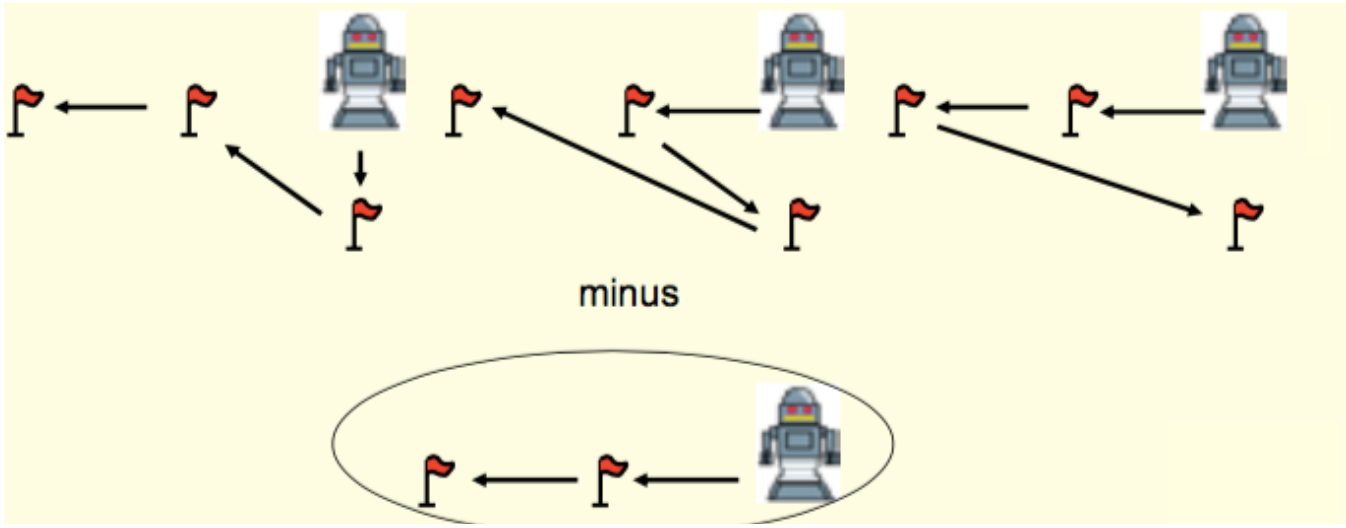
E.g., Team performance: (1+2+3+4+6+9+10+1+4+...)/22 = 5.8



- **Bid** the increase in the minimal sum of arrival times the robot needs from its current location to visit all targets it has won *if* it wins the target it is bidding on (***BidAvePath***)
- This bid automatically balances the avg of the arrival times of all team robots, without the need for knowing about their bids

Computing Optimal Bids is *NP-Hard*

- **Minimal path cost over a set of assigned tasks → *TSP !***
  - For large sets of tasks it might be *intractable* ☹
  - Even for small subsets, if bidding has to be done online in real-time, optimal computation of bids might be unfeasible
  - A number of heuristics: *2-Opt, 3-Opt, ACO, NN, Insertion heuristics*
- ***Full task allocation problem: Multi-robot routing → VRP***

Minimum (

…)

Complexity of auction mechanisms

- **Time complexity** (amount of computation)
  - (Distributed) Bid computations, in a single auction
  - + Winner determination, in a single auction
  - + Number of auctions required to assign all tasks
- **Communication complexity** (bandwidth for information exchanges)
  - Call for bids from the auctioneer
  - + Bids submission from the agents to the auctioneer
  - + Awarding tasks to winners (may or may not inform losers in addition to winners)

Solution Quality (team cost) → It depends whether the above complexity it allows to deal with all subproblems in *optimal* way or not

Time Complexity

Auction type	Bid computation	Winner determination	Number of auctions
Single-item	v	$O(r)$	n
Multi-item (greedy)	$O(nv)$	$O(nrm)$	$\lceil n/m \rceil$
Multi-item (optimal)	$O(nv)$	$O(nr^2)$	$\lceil n/m \rceil$
Combinatorial	$O(V \cdot 2^n)$	$O\big((b+n)^n\big)$	1

n = # of items
r = # of bidders
b = # of submitted bid bundles (combinatorial auctions)
m = max # of awards per auction (multi-item auctions), $1 \leq m \leq r$
v = item valuation (domain /<u>performance criterion dependent</u>)
V = bundle valuation (domain / <u>performance criterion dependent</u>)

Results from:

- [Gerkey and Matarić, IJRR 23(9), 2004]
- [Sandholm, Artificial Intelligence 135(1), 2002]

Communication Complexity

Auction type	Auction call (from auctioneer)	Bid submission (from agents)	Task awards (from auctioneer)	Task awards (+ ¬awarded)
Single—item	$O(r)$	$O(r)$	$O(1)$	$O(r)$
Multi-item	$O(rn)$	$O(rn)$	$O(m)$	$O(r)$
Combinatorial	$O(rn)$	$O(r \cdot 2^n)$	$O(n)$	$O(r+n)$

$n = \#$ of items
$r = \#$ of bidders
$m =$ max $\#$ of awards per auction (multi-item auctions), $1 \leq m \leq r$

Optimal Task Allocation: VRP

$$\min_{\boldsymbol{x}} \sum_{i \in V_T \cup V_R,\, j \in V_T} c_{ij} x_{ij}$$

Performance function (for MiniSum)

$$s.t.$$

$$\sum_{i \in V_T \cup V_R} x_{ij} = 1 \quad \forall j \in V_T$$

Each target vertex is entered exactly once

$$\sum_{j \in V_T} x_{ij} \leq 1 \quad \forall i \in V_T \cup V_R$$

Each (robot or target) vertex is left at most once

$$\sum_{i,j \in U} x_{ij} \leq |U| - 1 \quad \forall U \subseteq V_T : |U| \geq 2$$

Sub-tour elimination

V_R = Set of robot vertices
V_T = Set of task / target vertices
Variables: x_{ij} = edge (i, j) is in the solution
c_{ij} = Path cost from vertex i to vertex j

Note: this is referred to as a MIP (Mixed Integer Programming model) in the following

Worst-Case deviation from optimality

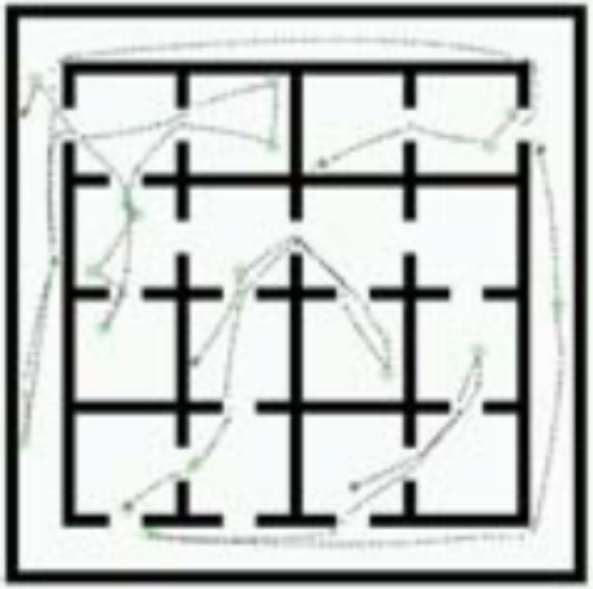
When using the mentioned bidding rules in sequential auctions, it is possible to compute the worst-case loss bounds (lower and upper) with respect to the optimal allocation computed using the optimization model (e.g., 1.5 means that using the BidSumPath bidding rule to optimize the MiniSum criterion, in the worst-case the team cost is at least 1.5 times the optimal team cost)

Bidding rule	Team Performance Criterion					
	MiniSum Lower - Upper		*MiniMax* Lower – Upper		*MiniAvg* Lower - Upper	
BidSumPath	1.5	2	n	$2n$	$\frac{m+1}{2}$	$2m$
BidMaxPath	n	$2n$	$\frac{n+1}{2}$	$2n$	$\Omega(m^{1/3})$	$2m$
BidAvgPath	m	$2m^2$	$\frac{n+1}{2}$	$2m^2n$	$\Omega(m^{1/3})$	$2m^2$

Experimental results for sequential auctions

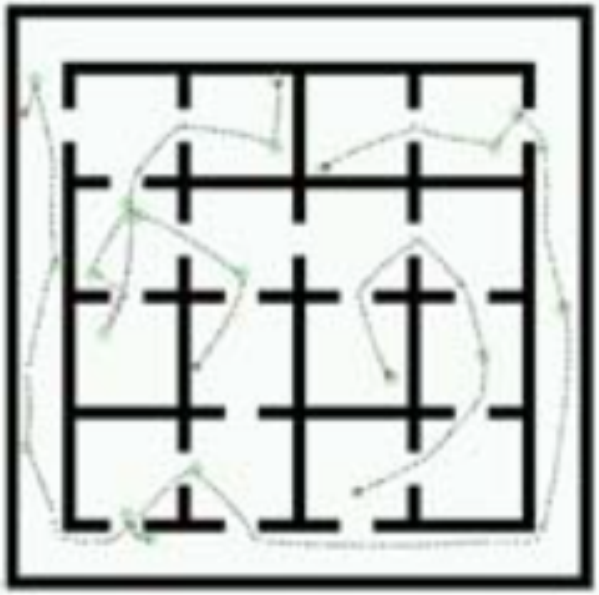
- 3 robots and 10 unclustered targets
- known terrain of size 51×51

parallel auctions



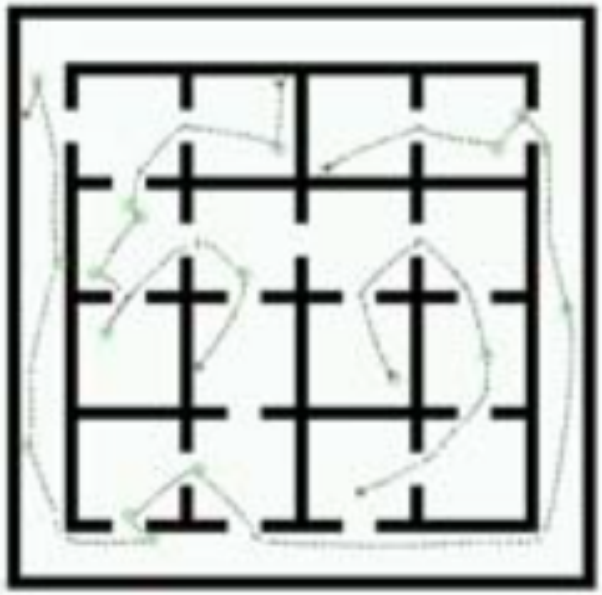
SUM = 426.98

sequential auctions



SUM = 279.62

optimal (MIP)
= ideal combinatorial auctions



SUM = 271.04

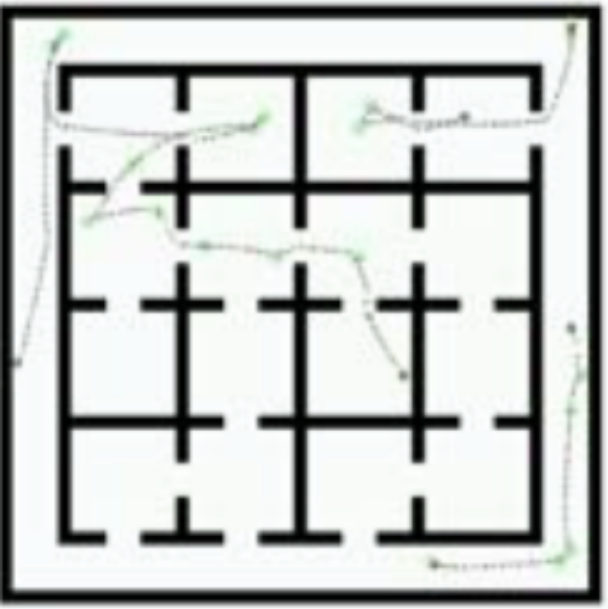
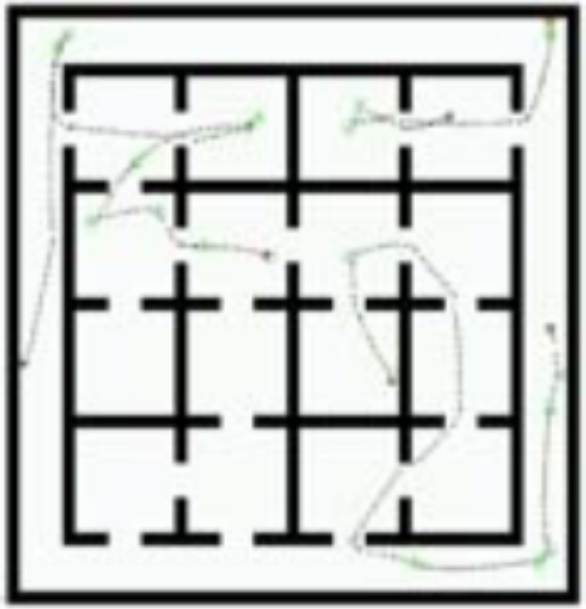
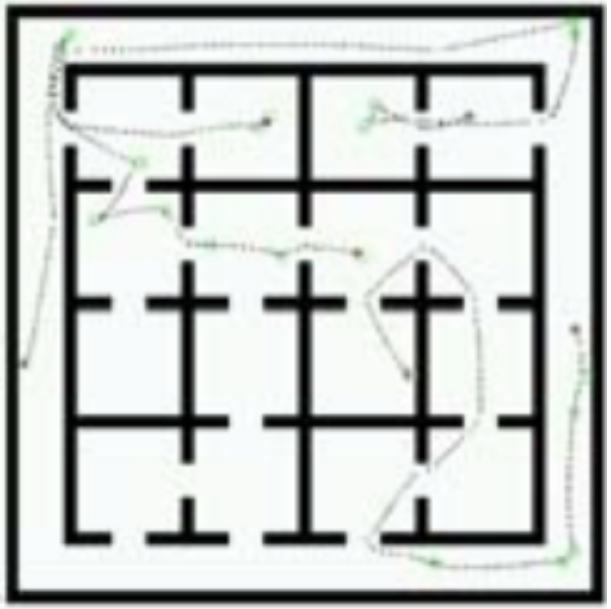
Experimental results for sequential auctions

- 3 robots and 10 unclustered targets
- known terrain of size 51×51

	SUM	MAX	AVE
BidSumPath	193.50	168.50	79.21
BidMaxPath	219.15	125.84	61.39
BidAvePath	219.16	128.45	59.12
optimal (MIP) = ideal combinatorial auctions	189.15	109.34	55.45

Experimental results for sequential auctions

Impact of different bidding rules

BidSumPath (for energy)	BidMaxPath (for makespan)	BidAvePath (for flowtime)
		
SUM = 182.50 MAX = 113.36 AVE = 48.61	SUM = 218.12 MAX = 93.87 AVE = 46.01	SUM = 269.27 MAX = 109.39 AVE = 45.15

Task Allocation Summary

- Task Allocation as a model for coordination, division of labor, role assignment in multi-agent/robot systems
- General formalization and taxonomy of multi-robot task allocation (MRTA) problems
- Optimization models for different classes of TA problems
- Computational complexity of the different classes / models
- Basic solution approaches exploiting the optimization models
- Intentional vs. Emergent task allocation
- Distributed approaches
- Stigmergy-based (emergent) methods
- Market-based methods: auction models, properties of different auction models (parallel, combinatorial, sequential), complexity