15-110: Principles of Computing

Homework 05

Due: 10th October, 2020 at 23:59

- You must solve the tasks **individually**.
- There are 50 points.

1. (15 points) **Exchanging Cards**

   Aliaa and Buthaina collect Yu-Gi-Oh cards, and they have some repeated cards they would like to trade. Aliaa and Buthaina are meeting today and they are both taking *one copy* of each repeated card they have. They would like to trade as many cards as possible, but they are not interested in cards they already have, of course.

   Your job is to write a program that computes the maximum number of cards that can be traded. To make your life easier, each card is identified by a unique number.

   Implement the function `exchangingCards(ca, cb)` that takes as input two lists: `ca`, Aliaa's repeated cards (all distinct); and `cb`, Buthaina's repeated cards (all distinct). The function returns the maximum number of cards they could exchange.

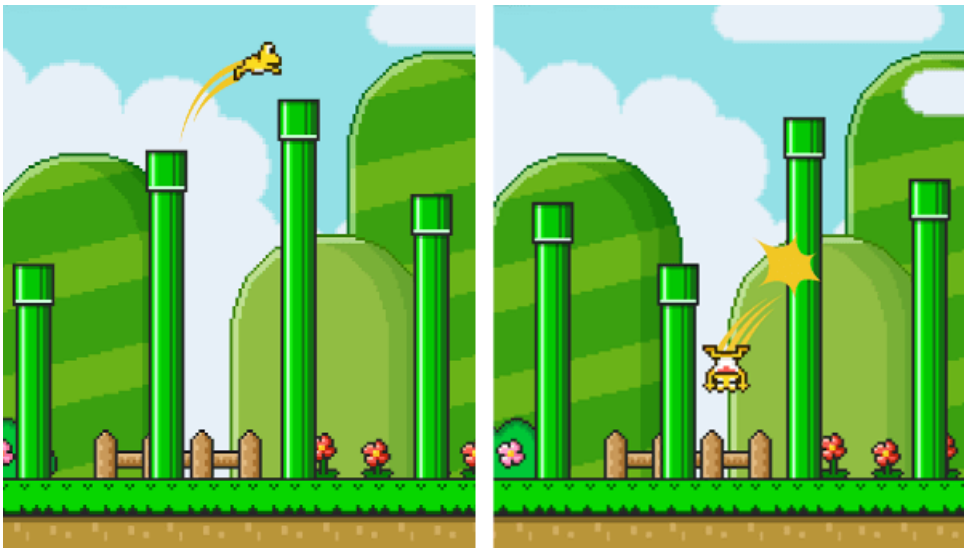
   For example:

   - `exchangingCards([1,2], [5,3,4])` should return 2
   - `exchangingCards([1,2], [5,2,1])` should return 0

2. (10 points) **Jumping Frog**

   In each stage of the Jumping Frog game you must safely get your amphibian through a sequence of pipes of different heights to the rightmost pipe. Nevertheless the frog only survives if the height difference of consecutive pipes is at most the frog jump/fall height. If the next pipe height is too high, the frog hits the pipe and falls. If the next pipe height is too low, the frog does not survive the fall. The frog always starts on the top of the leftmost pipe.

   In this game the distance of pipes is irrelevant, which means that the frog always can reach the next pipe with a jump.



Implement the function `jumpingFrog(P, h)` that takes as input a list `P` with the pipes' height (from left to right), and the frog's maximum jump/fall height `h`. The function should return `"Frog wins"` if the frog can reach the last pipe safely, and `"Game over"` if it cannot.

For example:

- `jumpingFrog([1,3], 2) == "Frog wins"`
- `jumpingFrog([4,5,2], 2) == "Game over"`

You may assume that `P` will have at least one element.

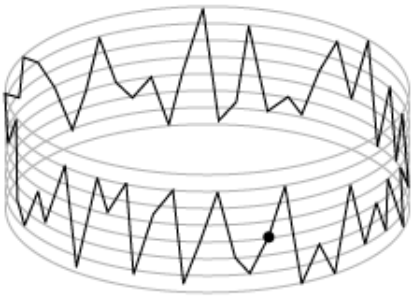
3. (10 points) **Musical Loop**

A musical loop is a small section of music composed to be played continuously (that is, the section is played again when it reaches the end), in a seamless way. Loops are used in many styles of popular music (hip hop, techno, etc), as well in computer games, especially casual games on the internet.

Loops may be digitalized for example using PCM (Pulse Code Modulation), a technique for representing analog signals used extensively in digital audio. In PCM, the magnitude of the signal is sampled at regular intervals, and the values sampled are stored in sequence. To produce the sound for the sampled data, the procedure is applied in reverse (demodulation).

Sideeg works for a game software house, and composed a beautiful musical loop, coded in PCM. Analyzing the waveform of his loop in audio editing software, Sideeg became curious when he noticed a number of "peaks". A peak in a waveform is a *local maximum or minimum*. The figure below illustrates (a) a waveform and (b) the loop formed with this waveform, containing 48 peaks.

(a) A waveform



(b) The same waveform as a loop

Given a sequence of samples in a musical loop, Sideeg wants to know how many peaks exist in the loop.

Implement the function `musicalLoop(L)` that takes as input a list of samples of the music composed by Sideeg, and return the total number of peaks that exist in that musical loop.

For example:

- `musicalLoop([1,-3]) == 2` (one local maximum and one local minimum)
- `musicalLoop([40,0,-41,0,41,42]) == 2` (one local minumum at -41, and one local maximum at 42)

You may assume that there are no consecutive samples of the same magnitude.

4. (15 points) **Prime Factors**

   The fundamental theorem of arithmetic states that every integer greater than 1 is the product of a unique combination of prime numbers.

   Implement the function `primeFactors(n)` that returns a list with all *distinct* prime factors of `n` in increasing order. If the number is less than 1, the function returns the empty list.

   For example:

   - `primeFactors(18) == [2,3]`
   - `primeFactors(5) == [5]`
   - `primeFactors(100) == [2,5]`