Final Solutions

15-122 Principles of Imperative Computation

Monday 4th May, 2015

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Recitation Section: S

Instructions

- This exam is closed-book with one sheet of notes permitted.
- You have 180 minutes to complete the exam.
- There are 7 problems on 23 pages (including 0 blank pages at the end).
- Use a **dark** pen or pencil to write your answers.
- Read each problem carefully before attempting to solve it.
- Do not spend too much time on any one problem.
- Consider if you might want to skip a problem on a first pass and return to it later.

	Max	Score
Union Find [C0]	40	
Grab Bag [C]	30	
Circular Queues	20	
Strings [C]	40	
Clac Revisited [C]	40	
Spanning Trees [C]	40	
I can C clearly now [C]	40	
Total:	250	

1 Union Find [C0] (40 points)

In this question, we consider a C0 implementation of the union find data structure from class. This is the more efficient version that stores tree heights, but without path compression.

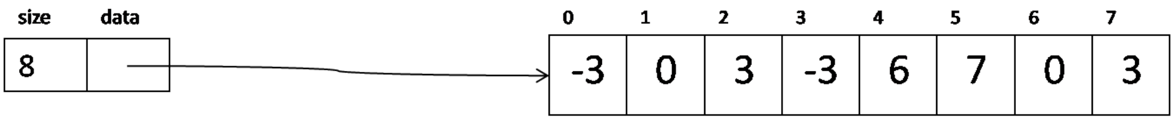
```
typedef struct ufs_header* ufs;
struct ufs_header {
  int size;
  int[] data;
};

bool is_ufs(ufs U) {
  return U != NULL && is_arr_expected_length(U->data, U->size);
}
```

10pts

Task 1 The `is_ufs` data structure invariant above is not sufficient to ensure the correctness of `ufs_find`.

```
int ufs_find(ufs U, int x)
//@requires is_ufs(U);
//@requires 0 <= x && x < U->size;
{
    int i = x;
    while (U->data[i] >= 0) {
        i = U->data[i];
    }
    return i;
}
```



If `U` contains the address of the data structure above, `ufs_find(U, 4)` will return

0

Show a `U` such that `is_ufs(U)` holds but `ufs_find(U, 4)` will cause a memory error:

(many answers possible)

Show a `U` such that `is_ufs(U)` holds but `ufs_find(U, 4)` will never terminate:

(many answers possible)

5pts **Task 2** In order to ensure that `ufs_find` can never cause a memory error, we need to extend the data structure invariant to check that every integer in the array `U->data` is…

…less than `U->size` (or, equivalently, less than `\length(U->data)`)

10pts **Task 3** Give loop invariant(s) for `ufs_find` that would ensure the array accesses on lines 6 and 7 are safe. Using the data structure invariant improvement described in Task 2, you should be able to reason that this loop invariant is true initially and preserved by an arbitrary iteration of the loop. You don't necessarily have to use all the lines.

```
//@loop_invariant ____________0 <= i && i < U->size____________;

//@loop_invariant _________________________________________________;

//@loop_invariant _________________________________________________;
```

10pts **Task 4** The *union* operation of union-find uses `ufs_find`:

```
1  void ufs_union(ufs U, int x, int y)
2  //@requires is_ufs(U);
3  //@ensures is_ufs(U);
4  {
5    int[] A = U->data;
6    int i = ufs_find(U, x);
7    int j = ufs_find(U, y);
8    //@assert A[i] < 0 && A[j] < 0;
9
10   if (i == j) return;
11   else if (A[i] == A[j]) { (A[i])--; A[j] = i; }
12   else if (A[i] < A[j]) { A[j] = i; }
13   else { A[i] = j; }
14 }
```

What postconditions does `ufs_find` need in order for us to reason that the assertion on line 8 always returns true – even if we know nothing about the implementation of `ufs_find`? You don't necessarily have to use all lines.

```
//@ensures _0 <= \result&& \result< U->size && U->data[\result] < 0_;

//@ensures ________________________________________________________;

//@ensures ________________________________________________________;
```

5pts **Task 5** In this implementation, if `U->data[0] == -9`, then that means `U->size` must be at least…

$256 = 2^{(9-1)}$

2 Grab Bag [C] (30 points)

6pts

Task 1 The following run times were obtained when using two different algorithms on a data set of size n. You are asked to determine asymptotic complexity of the algorithms based on this time data. Determine the asymptotic complexity of each algorithm as a function of n. Use big-O notation in its tightest, simplest form.

n	Execution Time
1000	0.564 milliseconds
2000	2.271 milliseconds
4000	8.992 milliseconds
8000	36.150 milliseconds

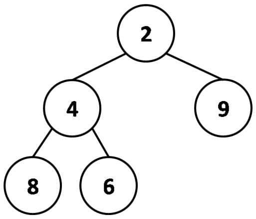
$O(\underline{\quad n^2 \quad})$

n	Execution Time
1000	0.042 milliseconds
1000000	0.042 milliseconds
1000000000	0.042 milliseconds

$O(\underline{\quad 1 \quad})$

6pts

Task 2



Represent the min-heap pictured above as an array by the method discussed in class:

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	2	4	9	8	6										

If the number 5 is added to this heap structure, what will the array look like afterwards?

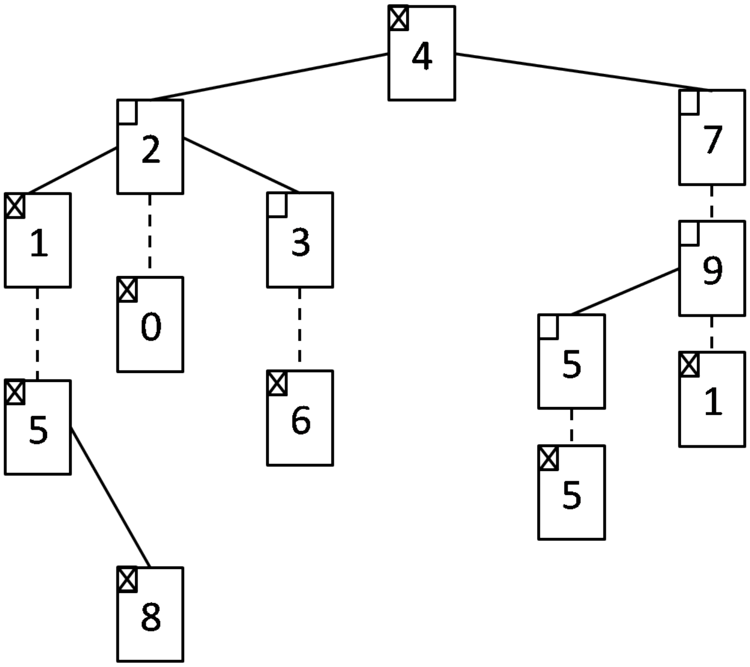
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	2	4	5	8	6	9									

If, *after* adding the number 5, we *then* remove the minimum element from the heap, what will the final contents of the array be?

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	4	6	5	8	9										

8pts **Task 3** **Note: 15-122 stopped covering tries in Spring 2015.**

The ternary search trie (TST) pictured below stores a set of numerical strings. List all numerical strings stored in the trie.



1, 15, 18, 20, 36, 4, 755, 791

10pts **Task 4** Consider the following C function that computes the integer square root of n, $n > 0$. The integer square root i of a positive integer n is the value of i such that $i^2 \leq n$ and $(i+1)^2 > n$. For example, isqrt(15) = 3 and isqrt(16) = 4.

```c
1  int isqrt(int n) {
2    REQUIRES(n < 46000);
3
4    if (n == 1) return 1;
5    int lower = 0;
6    int upper = 1 + n/2;
7
8    while(lower + 1 < upper) {
9      printf("  lower = %d, upper = %d\n", lower, upper);
10     int mid = lower + (upper - lower)/2;
11     int square = mid * mid;
12     if (square == n) {
13       lower = mid;
14       break;   // break out of loop, go straight to line 21
15     } else if (square < n) {
16       lower = mid;
17     } else {
18       upper = mid;
19     }
20   }
21   printf("  lower = %d, upper = %d\n", lower, upper);
22   return lower;
23 }
```

Trace the function for the following values of `n`, showing the values printed for `lower` and `upper` at the start of each iteration, along with their values after the loop terminates. You may not need to use all the available space provided. The first values output for `lower` and `upper` are given for you.

$n = 25$

lower	0	0	3	4	5			
upper	13	6	6	6	6			

$n = 42$

lower	0	0	5	5	6	6		
upper	22	11	11	8	8	7		

What is the worst case runtime complexity of the function above in terms of n using big O notation in its simplest, tighest form?

$O(\underline{\quad\quad \log n \quad\quad})$

3 Circular Queues (20 points)

Consider the following implementation of a bounded queue of integers in C:

```
typedef struct queue_header queue;
struct queue_header {
    int capacity;   // maximum size of queue (overflow not allowed)
    int front;      // index of front element of queue
    int rear;       // index of rear element of queue
    int *data;      // queue data, length of the array is capacity
};
```

An empty queue is always represented by `front` = -1 and `rear` = -1. Otherwise, front is the index of the first element of the queue and rear is the index of the last element of the queue. If an element is enqueued on to an empty queue, it is always stored at index 0 of the array. Elements of the queue are stored in contiguous cells of the array. Note that front index can be greater than rear index. In this case, the queue wraps around from the end of the array back to the beginning, as shown in the example below.

Q = queue_new(5);	☒	☒	☒	☒	☒	front = -1,	rear = -1,	capacity = 5
enq(Q, 4);	4	☒	☒	☒	☒	front = 0,	rear = 0,	capacity = 5
enq(Q, 7);	4	7	☒	☒	☒	front = 0,	rear = 1,	capacity = 5
enq(Q, 8);	4	7	8	☒	☒	front = 0,	rear = 2,	capacity = 5
deq(Q);	☒	7	8	☒	☒	front = 1,	rear = 2,	capacity = 5
deq(Q);	☒	☒	8	☒	☒	front = 2,	rear = 2,	capacity = 5
enq(Q, 9);	☒	☒	8	9	☒	front = 2,	rear = 3,	capacity = 5
enq(Q, 6);	☒	☒	8	9	6	front = 2,	rear = 4,	capacity = 5
enq(Q, 3);	3	☒	8	9	6	front = 2,	rear = 0,	capacity = 5
deq(Q);	3	☒	☒	9	6	front = 3,	rear = 0,	capacity = 5
deq(Q);	3	☒	☒	☒	6	front = 4,	rear = 0,	capacity = 5
enq(Q, 10);	3	10	☒	☒	6	front = 4,	rear = 1,	capacity = 5
deq(Q);	3	10	☒	☒	☒	front = 0,	rear = 1,	capacity = 5
deq(Q);	☒	10	☒	☒	☒	front = 1,	rear = 1,	capacity = 5
enq(Q, 42)	☒	10	42	☒	☒	front = 1,	rear = 2,	capacity = 5
deq(Q);	☒	☒	42	☒	☒	front = 2,	rear = 2,	capacity = 5
deq(Q);	☒	☒	☒	☒	☒	front = -1,	rear = -1,	capacity = 5
enq(Q, 2);	2	☒	☒	☒	☒	front = 0,	rear = 0,	capacity = 5

Assume there is a function `is_queue` that tests the data structure invariant for a `queue*` as described and illustrated above. Complete the following functions to implement the queue described above. All functions should run in constant time.

```
queue *queue_new(int n) {
    REQUIRES(n > 0);

    queue *Q = xmalloc(sizeof(struct queue_header));

    Q->data = xcalloc(n, sizeof(int));

    Q->front = -1;
    Q->rear = -1;
    Q->capacity = n;
    ENSURES(is_queue(Q) && queue_empty(Q));
    return Q;
}

void queue_free(queue *Q) {
    REQUIRES(is_queue(Q));

    free(Q->data);

    free(Q);
}

bool queue_empty(queue *Q) {
   REQUIRES(is_queue(Q));
   return Q->front == -1 && Q->rear == -1;
}

bool queue_full(queue *Q) {
   REQUIRES(is_queue(Q));

   return (Q->rear + 1) % Q->capacity == Q->front;
}

void enq(queue *Q, int element) {
   REQUIRES(is_queue(Q) && !queue_full(Q));

   if (Q->front == -1) Q->front = 0;

   if (Q->rear == -1 ) Q->rear = 0;

   else Q->rear = (Q->rear + 1) % Q->capacity;
   Q->data[Q->rear] = element;
   ENSURES(is_queue(Q));
}
```

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```
int deq(queue *Q) {
    REQUIRES(is_queue(Q) && !queue_empty(Q));

    int element = Q->data[Q->front];

    if (____________________Q->front == Q->rear____________________) {

       Q->front = ____________________-1____________________;
       Q->rear = -1;

    } else {

       Q->front = ____________(Q->front + 1) % Q->capacity____________;
    }

    ENSURES(is_queue(Q));
    return element;
}
```

4 Strings [C] (40 points)

20pts **Task 1** String buffers are useful as a data structure because concatenating strings naively can get really expensive. Concatenating a string of length x and a string of length y with the library function `strcat` requires $x + y$ constant-time operations.

(In your answers below, both n and k can vary: don't treat k as a constant.)

If we are concatenating n strings with C's `strcat`, where each string has length k, by joining them one at a time (so we have n strings of length k, then $n - 2$ strings of length k and 1 string of length $2k$, and then we have $n - 3$ strings of length k and one string of length $3k$…), the *total* running time of the process taking n length-k strings and returning a single string of length nk will be in

$O(\underline{\qquad\qquad kn^2 \qquad\qquad})$

```
abc   def   ghi   jkl   mno   pqr   stu   vwx
abcdef      ghi   jkl   mno   pqr   stu   vwx
abcdefghi         jkl   mno   pqr   stu   vwx
abcdefghijkl            mno   pqr   stu   vwx
abcdefghijklmno               pqr   stu   vwx
abcdefghijklmnopqr                  stu   vwx
abcdefghijklmnopqrstu                     vwx
abcdefghijklmnopqrstuvwx
```

If we instead only concatenate pairs of strings with equal length (so we have n strings of length k, then $n/2$ strings of length $2k$, and then $n/4$ strings of length $4k$), the *total* running time of the process taking n length-k strings and returning a single string of length nk will be in

$O(\underline{\qquad\qquad kn \log n \qquad\qquad})$

```
abc   def   ghi   jkl   mno   pqr   stu   vwx
abcdef      ghi   jkl   mno   pqr   stu   vwx
abcdef      ghijkl      mno   pqr   stu   vwx
abcdef      ghijkl      mnopqr      stu   vwx
abcdef      ghijkl      mnopqr      stuvwx
abcdefghijkl            mnopqr      stuvwx
abcdefghijkl            mnopqrstuvwx
abcdefghijklmnopqrstuvwx
```

If we use a single correctly-implemented string buffer to add the n strings of length k to the buffer one at a time, and then we use `strbuf_str` to make a copy of the final string with size nk, the running time of this whole process will be in

$O(\underline{\qquad kn \qquad})$

During the process described immediately above, the *worst-case* running time of a *single* call to the function `strbuf_addstr` could be in

$O(\underline{\qquad kn \qquad})$

Despite this, our amortized analysis of the problem ensures *most* calls to `strbuf_addstr` will have a running time that is in

$O(\underline{\qquad k \qquad})$

16pts **Task 2** For these questions, imagine that we have placed n Andrew IDs, represented as strings of 2-8 characters, into a separate-chanining hashtable with a capacity (table size) of m, where n and m are both very large. (Don't assume anything about their relationship, though: n could be much larger than m or vice-versa.)

If our hash function takes `s` and returns `strlen(s) * 1664525 + 1013904223`, where `strlen` gives us the length of the Andrew ID we expect that a single lookup or insertion to take time in

$O(\underline{\qquad n \qquad})$

If our hash function always returns 4, we expect that a single lookup or insertion to take time in

$O(\underline{\qquad n \qquad})$

If we know the n Andrew IDs in advance, then the best possible hash function for those andrew IDs would ensure that a single lookup or insertion takes time in

$O(\underline{\qquad n/m \qquad})$

Is the pseudorandom number generator `hash_lcg` discussed in lab and lecture, which applies a linear congruential generator to every character in the string, always going to ensure this best possible performance? *Briefly* justify your answer (a sentence or two at most).
(Hint: The specifics of `hash_lcg` aren't important here, it's just an example of a good hash function.)

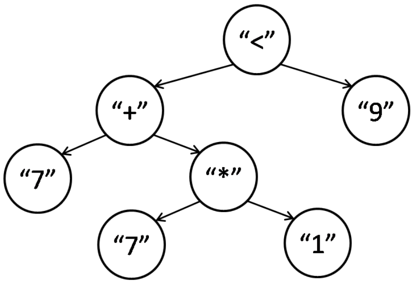
No: it's possible to create collisions with any hash function.

4pts **Task 3** If a hashtable using open addressing (specifically linear probing) contains k elements and has a table with size k, then looking up any key that is not in the table will take time in

$O(\underline{\qquad\qquad k \qquad\qquad})$

5 Clac Revisited [C] (40 points)

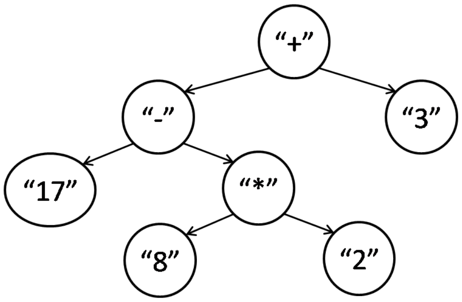
One of the jobs of a parser is to take an infix expression like `7 + 7 * 1 < 9` and figure out that it is supposed to be understood as `(7 + (7 * 1)) < 9`. This unambiguous representation of the structure of an expression can be represented as a tree structure:



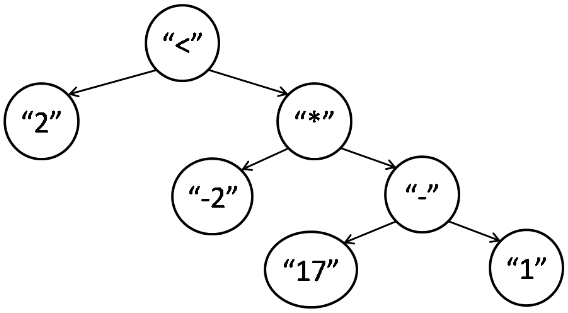
Clac gave us another way to unambiguously represent expressions without using parentheses. We can represent the expression given as a tree above in Clac by writing `7 7 1 * + 9 <`.

Before				After			
Stack		**Queue**		**Stack**		**Queue**	**Cond**
S	$\|\|$	n, Q	$\longrightarrow$	S, n	$\|\|$	Q	
S, x, y	$\|\|$	`+`$, Q$	$\longrightarrow$	$S, x + y$	$\|\|$	Q	
S, x, y	$\|\|$	`-`$, Q$	$\longrightarrow$	$S, x - y$	$\|\|$	Q	
S, x, y	$\|\|$	$*, Q$	$\longrightarrow$	$S, x * y$	$\|\|$	Q	
S, x, y	$\|\|$	`<`$, Q$	$\longrightarrow$	$S, 1$	$\|\|$	Q	if $x < y$
S, x, y	$\|\|$	`<`$, Q$	$\longrightarrow$	$S, 0$	$\|\|$	Q	if $x \geq y$

5pts **Task 1** Draw the tree corresponding to the C0 expression `(17 - 8*2 + 3)`, which evaluates to 4.



5pts **Task 2** Draw the tree corresponding to the Clac expression `2 -2 17 1 - * <`.



15pts **Task 3** Expression trees can be written in C as elements of the type `exp*`. The specification function `is_binop` checks that a `char*` is a well-formed C string representing a binary operator: either `"<"`, `"*"`, `"+"`, or `"-"`. The specification function `is_int` checks that a `char*` is a well-formed C string that can be parsed as a 32-bit signed integer.

```
typedef struct exp_node exp;
struct exp_node {
  char *data;
  exp *left;
  exp *right;
};
bool is_exp(exp *E) {
  if (E == NULL) return false;
  if (E->left == NULL && E->right == NULL && is_int(E->data)) return true;
  return is_exp(E->left) && is_exp(E->right) && is_binop(E->data);
}
```

Clac programs are represented by queues of strings: we represent the Clac program `2 -2 17 1 - * <` as a queue with `"2"` is at the front of the queue and `"<"` at the back of the queue. Strings are pointers to `char` in C, so generic queues can easily hold strings.

```
bool queue_empty(queue_t Q);
void enq(queue_t Q, void *x);
void *deq(queue_t Q); // Requires !queue_empty(Q);
```

Write a (simple and efficient!) recursive procedure for converting a `exp*` expression into a Clac program and adding that program to the end of a queue. Don't modify the existing tree `E`, don't call the specification functions `is_int` and `is_binop`, and don't explicitly allocate or free memory (calling `enq` and `deq` is fine, though). Explicitly write a cast whenever you convert pointers.

```
void convert(exp *E, queue_t Q) {
  REQUIRES(is_exp(E));

  if (E->left != NULL) {
    convert(E->left, Q);
    convert(E->right, Q);
    enq(Q, (void*)E->data);
  } else {
    enq(Q, (void*)E->data);
  }

}
```

15pts **Task 4** Clac and the C0VM behave in essentially the same way, except that one is based on queues and the other is based on a program counter that can move around more freely. Here are some bytecode instructions you'll use in this question (you don't need to use them all):

```
0x60 iadd             S, x:w32, y:w32 -> S, x+y:w32
0x7E iand             S, x:w32, y:w32 -> S, x&y:w32
0x6C idiv             S, x:w32, y:w32 -> S, x/y:w32
0x68 imul             S, x:w32, y:w32 -> S, x*y:w32
0x64 isub             S, x:w32, y:w32 -> S, x-y:w32
0x10 bipush <b>       S               -> S, x:w32 (x = (w32)b, sign extended)
0x00 nop              S               -> S
0xA1 if_icmplt <o1,o2> S, x:w32, y:w32 -> S     (pc = pc+(o1<<8|o2) if x < y)
0xA7 goto <o1,o2>     S               -> S     (pc = pc+(o1<<8|o2))
0xB0 return           ., v            -> .     (return v to caller)
```

Write C0VM bytecode that behaves, as much as possible, the same way as the Clac code `2 -2 17 1 - * <`. Your code should return either 0 or 1.

You only have to write the direct bytecode ("`15 02`"), but you can also write the memonic forms ("`vload 2`"). You don't have to use every line.

```
# C0 bytecode that mimics the Clac code 2 -2 17 1 - * <

10 02           # bipush 2

10 FE           # bipush -2

10 11           # bipush 17

10 01           # bipush 1

64              # isub

68              # imul

A1 00 08        # if_cmplt 8

10 00           # bipush 0

A7 00 05        # goto 5

10 01           # bipush 1

B0              # return
```

6 Spanning Trees [C] (40 points)

The C interface for a *weighted* undirected graph with positive integer weights is given below:

```
typedef unsigned int vertex;

void graph_free(graph G);
unsigned int graph_size(graph G);  // Number of vertices in the graph
graph graph_new(unsigned int n);
  // New graph with n vertices and no edges, requires n > 0
bool graph_hasedge(graph G, vertex v, vertex w);
  // Requires v < graph_size(G) && w < graph_size(G);
void graph_addedge(graph G, vertex v, vertex w, int weight);
  // Requires v < graph_size(G) && w < graph_size(G);
  // Requires !graph_hasedge(G, v, w);
  // Requires weight > 0;
int graph_getweight(graph G, vertex v, vertex w);
  // Requires v < graph_size(G) && w < graph_size(G);
  // Requires graph_hasedge(G, v, w);
```

10pts **Task 1** Assume an ***adjacency list*** implementation of graphs, as indicated below.

```
typedef struct graph_header *graph;
typedef struct adjlist_node adjlist;
struct adjlist_node {
  vertex vert;
  int weight;  // weight on edge to vertex vert
  adjlist *next;
};
struct graph_header {
  unsigned int size;
  adjlist **adj; // An array of adjacency lists
};
```

Write the graph function `graph_getweight`, assuming the existence of `is_graph`.

```
int graph_getweight(graph G, vertex v, vertex w) {
  REQUIRES(is_graph(G) && v < graph_size(G) && w < graph_size(G));
  REQUIRES(graph_hasedge(G, v, w));

  adjlist* p = G->adj[v];
  while (p != NULL) {
    if (p->vert == w) return p->weight;
    p = p->next;
  }
  return -1;

}
```

Prim's algorithm is another way to compute a minimum spanning tree for a graph. In this implementation of Prim's algorithm, we need an array of booleans to mark when each vertex is added to the minimum spanning tree. We also need a priority queue to hold edges under consideration, ordered based on weight: the edge with the highest priority in the priority queue is the edge with minimum weight.
Assume the following C interface for priority queues.

```
typedef void *elem;
typedef bool higher_priority_fn(elem e1, elem e2);

// (*prior)(e1,e2) returns true if e1 is *strictly* higher priority than e2
pq_t pq_new(size_t capacity,            /* > 0 */
            higher_priority_fn *prior); /* != NULL */
bool pq_empty(pq_t P);
void pq_add(pq_t P, elem e);
elem pq_rem(pq_t P);         /* Must not be empty */
void pq_free(pq_t P);        /* Must be empty */
```

In the next two tasks, you will complete a client program to implement Prim's algorithm. Note that for these tasks, you should respect the interfaces for graphs and priority queues. From the client's perspective, you do not know how these data structures are implemented. Start with the following client code in `prim.c`:

```
#include <stdlib.h>
#include <stdbool.h>
#include "lib/graph.h"
#include "lib/pq.h"
#include "lib/xalloc.h"
#include "lib/contracts.h"

struct edge_header {  // edge from v to w
    vertex v;
    vertex w;
    int weight;
};
typedef struct edge_header* edge;
```

6pts **Task 2** First, write a client function `edgepriority` of type `higher_priority_fn` that can be used with this priority queue. This function will be stored in `prim.c` also.

```
bool edgepriority(elem e1, elem e2 ) {

      REQUIRES(             e1 != NULL && e2 != NULL                 );

      return        ((edge)e1)->weight < ((edge)e2)->weight        ;

}
```

20pts **Task 3** We will now write a function `prim` (in the same file `prim.c`) that will compute and return a new graph representing the minimum spanning tree of a graph G with at most 1000 vertices. Complete the missing parts.

```
graph prim(graph G) {
  REQUIRES(graph_size(G) <= 1000);
  // Requires that the graph G is connected

  unsigned int n = graph_size(G);

  // Create a new priority queue whose capacity should be large enough
  // to hold every edge of the graph.

  pq_t PQ = pq_new(__________n*n__________ , ____&edgepriority____);

  // Create the array of bool to mark each vertex that is
  // added to the minimum spanning tree, all set to false.

  bool *mark = _____________xcalloc(n, sizeof(bool))_____________;

  // Create a new graph for the minimum spanning tree
  // with the same number of vertices as G but no edges.

  graph T = graph_new(n);

  // Start with vertex 0 as the first vertex added to tree T
  vertex v = 0;
  mark[v] = true;
  unsigned int numv = 1;  // number of vertices in spanning tree

  // While the spanning tree is not complete...

  while (numv != _________________________n_________________________) {

    // For each neighbor w of v, if w is not in the spanning tree,
    // add the edge to the neighbor to the priority queue.
    for (vertex w = 0; w < n; w++) {

      if (________graph_hasedge(G,v,w) && !mark[w]________) {
         edge e = xmalloc(sizeof(struct edge_header));
         e->v = v;
         e->w = w;
         e->weight = graph_getweight(G,v,w);
         pq_add(PQ, e);
      }
    }
```

THE BODY OF THE WHILE LOOP IS CONTINUED ON NEXT PAGE...

Task 3 continued...

```
    // Retrieve the minimum weight edge from priority queue
    // until you find one that leads to an unmarked vertex.

    edge minedge = pq_rem(PQ);

    while (__________________mark[minedge->w]__________________) {
      free(minedge);
      minedge = pq_rem(PQ);
    }

    // Add this edge to the minimum spanning tree
    graph_addedge(T, minedge->v, minedge->w, minedge->weight);
    numv++;

    // Now consider edges from vertex w in the next iteration.
    v = minedge->w;
    mark[v] = true;
    free(minedge);
  }

  // Free up remaining dynamically-allocated memory.
  free(mark);
  while (!pq_empty(PQ)) {

    free(____________________pq_rem(PQ)____________________);
  }
  pq_free(PQ);

  // Return answer.

  return __________________________T__________________________;
}
```

4pts **Task 4** If our graph has v vertices and e edges, what is the worst case runtime complexity of any single call to `pq_rem` in the algorithm above if the priority queue is implemented using a heap? Express your answer using big O notation in its simplest, tightest form as a function of v and/or e.

$O(\underline{\qquad\qquad \log e \qquad\qquad})$

7 I can C clearly now [C] (40 points)

For all the parts of this question, you can assume standard implementation-defined behavior: 8-bit bytes, 2-byte shorts, and so on. In every case, assume we are calling `gcc` with the command

```
gcc -Wall -Wextra -Werror -Wshadow -std=c99 -pedantic example.c
```

20pts **Task 1** For each of the five following code examples, complete the given assignments in such a way that undefined behavior is triggered on the specified line (and **not before**). Only give values that are in range of the specified type.

If there is *no* way to trigger undefined behavior, check the box instead.

```
unsigned char i = ______________252, 253, 254, or 255______________;
char *S = xcalloc(63, sizeof(int));

// Cause undefined behavior on the next line:
char c = S[i] + 5;
```

Check this box if there is no way to trigger undefined behavior: ☐

```
unsigned int x = __________________________________________________;

signed short y = __________________________________________________;

unsigned int z = (unsigned int)(signed int)y;

// Cause undefined behavior on the next line:
if (x + z < x) y = 0xbad;
```

Check this box if there is no way to trigger undefined behavior: ☒

```
int x = ____0 /* array index verflows to SIZE_T_MAX */____;

assert(0 <= x);
int *A = xcalloc(100, sizeof(int));

// Cause undefined behavior in this for loop
for (size_t i = (size_t)(unsigned int) x; i < 100; i++)
    A[i] = A[i-1] * 2;
```

Check this box if there is no way to trigger undefined behavior: ☐

```
int i = ________/* Anything where i+j overflows */________;

int j = __________________________________________________;

int k = ____________/* for k is in (0,100000)*/___________;

assert(0 < k && k < 100000);
void **A = xmalloc(sizeof(void*) * k);

// Cause undefined behavior on the next line:
if (0 <= i + j && i + j < k) A[i + j] = NULL;
```

Check this box if there is no way to trigger undefined behavior: ☐

```
size_t x = __________________________________________________;

assert (x <= strlen("Hello"));
char *str = "Hello";

// Cause undefined behavior in this for loop
for (size_t i = 0; i < x; i++) {
    str = str + 1;
}
printf("The string is \"%s\"\n", str)
```

Check this box if there is no way to trigger undefined behavior: ☒

5pts **Task 2** Which of the following code snippets would cause undefined behavior? Circle `Error` if there is undefined behavior, otherwise circle `OK`.

```
void* x = xmalloc(sizeof(int));
int i = *(int*)x;
```

(Uninitialized read)

Circle one: [✓ Error] OK

```
void* x = xcalloc(1, sizeof(int));
int i = *(int*)x;
```

(4 byte allocation)

Circle one: Error [✓ Ok]

```
void* x = xcalloc(1, sizeof(short));
int i = *(int*)x;
```

(2 byte allocation)

Circle one: [✓ Error] OK

```
void* x = xcalloc(2, 3);
int i = *(int*)x;
```

(6 byte allocation)

Circle one: Error [✓ Ok]

```
void* x = xcalloc(sizeof(void*), 1);
int i = *(int*)x;
```

(8(4?) byte allocation)

Circle one: Error [✓ Ok]

12pts **Task 3** For each example, either say what `y` is at the end of the code snippet as an 8-hex-digit constant or write **undefined** if any undefined behavior occurs.

```
short w = -1;
unsigned short x = (unsigned short)w;
x = x << 4;
unsigned int y = (unsigned int)x;
```

y is 0x0000FFF0

```
signed char x = -2;
unsigned int y = (unsigned int)(int)x;
y = y << 8;
```

y is 0xFFFFFE00

```
unsigned char x = 3;
int y = (int)(signed char)x;
y = y << 32;
```

y is UNDEFINED

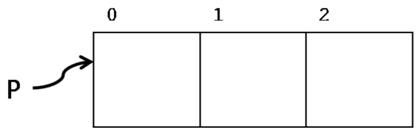
```
unsigned char x = 255;
x = x >> 1;
int y = (int)(signed char)x;
```

y is 0x0000007F

3pts **Task 4** The following function is intended to do a C0VM-like operation: treating an array of 3 bytes like a signed integer. Given the bytes `{80, 00, 00}`, for instance, this function returns the signed quantity `0xFF800000` (or -8388608), as it should.

Reveal the bug in this function by giving a test case, the desired outcome, and the actual result.

```
// Treats three bytes as a signed integer
int32_t g(uint8_t *P) {
  int32_t x = (int32_t)(int8_t)P[0];
  int32_t y = (int32_t)(int8_t)P[1];
  int32_t z = (int32_t)(int8_t)P[2];
  int32_t r = (x << 16) | (y << 8) | z;
  return r;
}
```



Desired: `0x`________________

Actual: `0x`________________

POSSIBLE SOLUTIONS: Anything that causes sign extension in a low-order byte and has something besides FF in a higher-order byte. Example: 00 FF FF