## Lab 14: Spend Some Cycles Thinking

Collaboration: In lab, we encourage collaboration and discussion as you work through the problems. These activities, like recitation, are meant to get you to review what we've learned, look at problems from a different perspective and allow you to ask questions about topics you don't understand. We encourage discussing problems with your neighbors as you work through this lab!
Setup: Copy the lab code from our public directory to your private directory:

```
cd private/15122
mkdir lab14
cd lab14
wget https://web2.qatar.cmu.edu/~srazak/courses/15122-s20/lab/handout-14.tgz
tar xfvz handout-14.tgz
```

You should add your code to the existing files graph.c, graph-search.c, graph-search.h, and graph-test.c.

Grading: Finish through (2.d) for full credit, and finish (3.a) and (3.b) for extra credit.

## The graph interface

This lab involves implementing a graph using an adjacency matrix rather than an array of adjacency lists. Graphs will be specified by the following C interface (as in graph.h):

```
typedef unsigned int vertex;
// typedef _-____* graph_t;
// typedef _-_-__* neighbors_t;
// New graph with v vertices
graph graph_new(unsigned int v);
//@ensures \result != NULL;
void graph_free(graph G);
//@requires G != NULL;
unsigned int graph_size(graph G);
//@requires G != NULL;
bool graph_hasedge(graph G,
    vertex v,
    vertex w);
//@requires G != NULL;
//@requires v < graph_size(G);
//@requires w < graph_size(G);
```

```
void graph_addedge(graph G, vertex v, vertex w);
//@requires G != NULL;
//@requires v != w;
//@requires v < graph_size(G);
//@ensures w < graph_size(G);
//@requires !graph_hasedge(G, v, w);
neighbors_t graph_get_neighbors(graph_t G, vertex v);
//@requires G != NULL && v < graph_size(G);
//@ensures \result != NULL;
bool graph_hasmore_neighbors(neighbors_t nbors);
//@requires nbors != NULL;
vertex graph_next_neighbor(neighbors_t nbors);
//@requires nbors != NULL;
//@requires graph_hasmore_neighbors(nbors);
void graph_free_neighbors(neighbors_t nbors);
//@requires nbors != NULL;
```


## Representing undirected graphs with an adjacency matrix

In class, we discussed the adjacency list implementation of graphs. In this lab, we'll work through the adjacency matrix implementation.
Recall that if a graph has $n$ vertices, then its adjacency matrix adj is an $n \times n$ array of booleans such that adj[i][j] is true if there is an edge from vertex $i$ to vertex $j$ (for valid $i$ and $j$ ), false otherwise. Since the graph is undirected, if adj[i][j] is true, then adj[j][i] should also be true, and if adj[i][j] is false, then adj[j][i] should also be false. The graph should not have any self-loops (i.e., a vertex with an edge to itself).
(2.a) Complete the data structure invariant function is_graph that returns true if G points to a valid graph given the definition above, or false otherwise.

Make sure to capture the fact that the graph is undirected in your data structure invariant! Compare notes with a neighbor before you move on.
(2.b) Complete the graph_new function that creates a new graph using a dynamically-allocated 2D array of boolean for the adjacency matrix. Create the 2D array in two steps: first create a new 1 D array of type bool*, then for each array element, have it point to a new 1 D array of type bool. You can then access the array using the 2 D notation (e.g., $G->a d j[0][1]=$ true).

Note: Don't ever do this in practice! C has ways of supporting 2D arrays that don't require an extra array of pointers; you'll learn about this more efficient way of doing things in later classes, like 15-213.
(2.c) Complete the functions graph_hasedge that checks if an edge is in the graph and graph_addedge that adds a new edge to the graph.
(2.d) Complete the graph_free function that frees any dynamically-allocated memory for the given graph G.

The functions graph_get_neighbors, graph_hasmore_neighbors, graph_next_neighbor and graph_free_neighbors have been pre-implemented for you at the very bottom of file graph.c, but for an extra challenge write them yourself.
Once you are done implementing the functions above, you should have a complete graph.c. Compile your code and test it with the given DFS and BFS searches in graph-search. c and the given graphs in graph-test.c:
\% make graphtest
\% ./graphtest
All tests should pass. (Look at the graphs in graph-test.c to see why.) Be sure to use valgrind

## Testing for graph connectedness

We say that a graph $G$ is connected if there is a path from any vertex to any other vertex in $G$. In an undirected graph, this definition is equivalent to saying that there is a path from a single arbitrary vertex to any other vertex. Can you see why?
(3.a) Write a function connected(G) in graph-search.c that returns true if a graph $G$ is connected, or false otherwise. Make sure your implementation is as efficient as possible.

Hint: Perform a BFS and count the number of vertices visited. For a connected graph, the total should be a specific value. Test your function on several graphs, connected and not connected.
(3.b) Update graph-search.h with the new function, and write at least two test cases in graph-test.c: one where connected returns true, and one where it returns false.

